DmitryWeise

Paper: **PHYLLOTACTIC REPRESENTATION OF CALENDARS**

Abstract:
Everywhere we observe the periodic phenomena, for example, change of day to night, seasons of year, etc. Usually for the image of these time phenomena use rectangular or circular tables (calendars, dial of hours). In the general words, rectangular tables are habitual for perception, they transfer originality of varying years, but graphically cycle form, i.e. a circle symbolizing repetition, is lost.
Circular tables are closed, and all periods appear similar one on another.
The spiral form, in a sense, is intermediate. It unites advantages and levels lacks of both mentioned above forms. However, the spiral table is not habitual and is not so easy for perception.

Often used structure for the description of an arrangement of leaves on plants (phyllotaxis) is the integer lattice in polar or cylindrical system of coordinates. From arithmetic positions in these lattices obvious spiral rows of the points numbered by the age represent residue classes modulo m. As a rule, modules on plants are Fibonacci numbers.
Modular arithmetic is the reliable tool for work with calendars. The generality of the methodical approach to periodicity in plants and in time phenomena has prompted new graphic images of the well known phenomena.
In the work some of possible constructions of periodic time processes in phyllotactic style are presented:

Chinese *calendar* 60-year cycle,
Maya calendar 260-year cycle,
Daaríisky Krugolét Chislobóga 144-year cycle,
Metonic 19-year lunar cycle,
Scandinavian lunar calendar,
Eternal calendar with possibility of definition of Easter day.

Chinese calendar *60-year cycle*

Maya calendar 260-year cycle

Topic: Design Approach

International Society for the Interdisciplinary Study of Symmetry (ISIS)
Russia

References:
[1] Roger V. Jean, "*Phyllotaxis: A Systematic Study in Plant Morphogenesis*", New York: Cambridge University Press, 1994

Contact:
phyllon@list.ru

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Phyllotactic representation of calendars

Dr D. Weise
International Society for the Interdisciplinary Study of Symmetry (ISIS), Moscow, Russia
e-mail: phyllon@list.ru

Premise

In botany the arrangement of repeated units such as leaves around a plant stem does satisfy the prevailing mathematical regularity properties. The generality of the methodical approach to periodicity in plants and in calendars has prompted new graphic images of the well known time phenomena.

1. Phyllotaxis

Phyllotaxis or phyllotaxy (Gr. *Phyllo* - leaf + *Taxis* - arrangement) is the study of the arrangement of repeated units such as leaves around a stem, scales on a pine cone or on a pineapple, florets in the head of a daisy, and seeds in a sunflower.

Most arrangements of leaves fall into 3 or 4 main categories: spiral, distichous, whorled, and multijugate. Mathematically, all these patterns are types of lattices.

Spiral arrangements are most frequent and they are classified by the number of spirals (parastichies) they exhibit.

In a spiral lattice, the eye tends to connect nearest points into spirals. These spirals are called parastichies.

The number of parastichies in spiral arrangements are most often Fibonacci numbers (1, 1, 2, 3, 5, 8, 13, 21 ...) and the angle between successive leaves is

close to the Golden Angle - about 137.5 degrees. This frequent type of pattern is called Fibonacci phyllotaxis. [1]

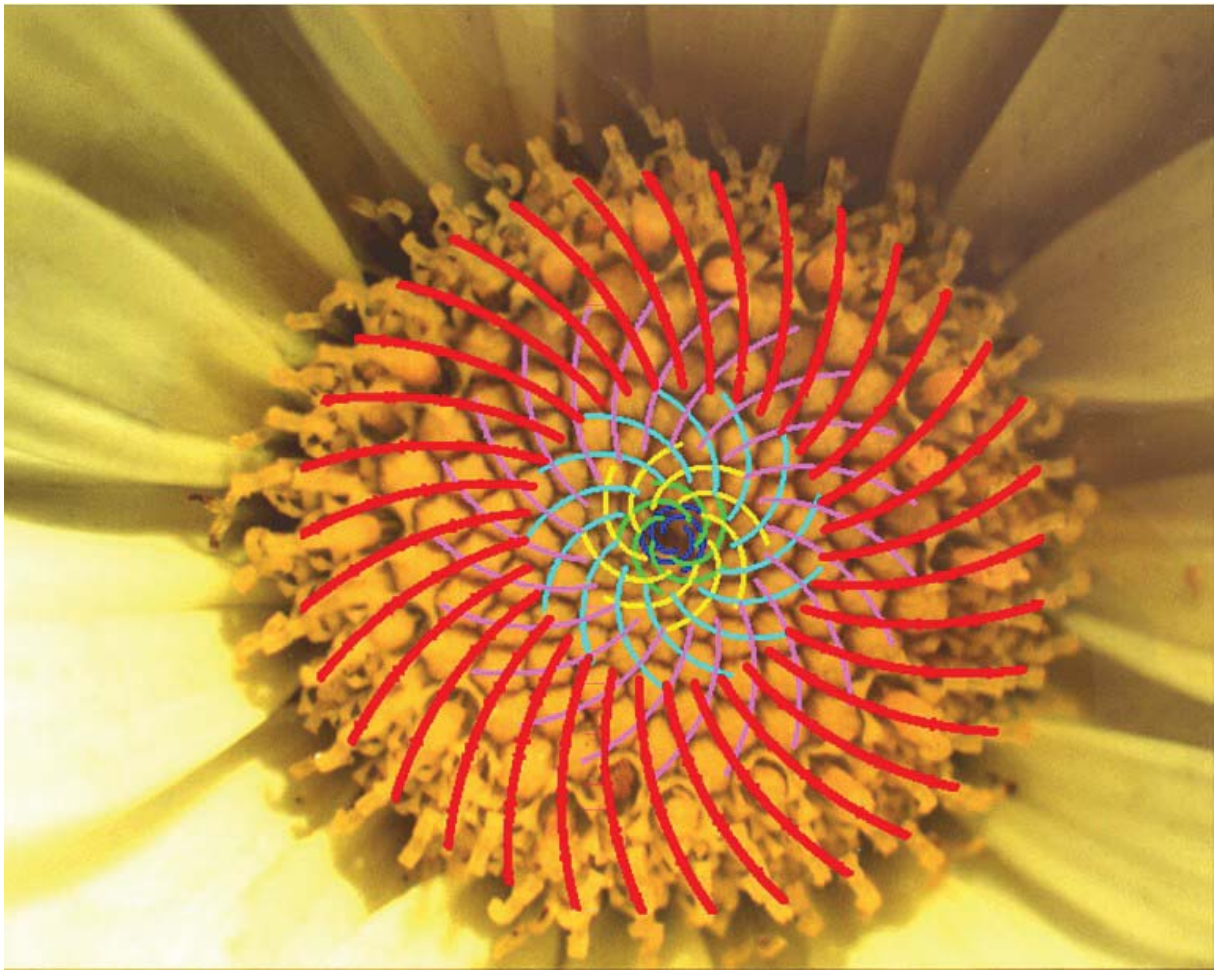

Fig. 1 Phyllotactic pattern on a Daisy

These spirals normally come in two *families*, yielding a pair of numbers, called parastichy numbers.

In 1837 brothers Bravais (Auguste and Louis) have opened, that numbers of the next leaves on a spiral correspond to quantity of spirals in family of spirals.

In terms of Modular Arithmetic the statement of brothers Bravais is equivalent to statement.

There are precisely m residue classes modulo m.

So numbers of leaves on a spiral represent *the residue class or the congruence class of a modulo*.

Each spiral is a residue class.

2. Calendars

Everywhere we observe the periodic phenomena, for example, change of day to night, seasons of year, etc. Usually for the image of these time phenomena we use rectangular or circular tables (calendars, dial of hours).

In the general words, rectangular tables are habitual for perception, they transfer originality of varying years, but graphically cycle form, i.e. a circle symbolizing repetition, is lost.

The circular tables are closed, and all periods appear similar to one another.

The spiral form, in a sense, is intermediate. It unites advantages and levels lacks of both forms mentioned above. However, the spiral table is not habitual and is not so easy for perception.

Modular arithmetic is the reliable tool for work with calendars.

2.1 Chinese calendar 60-year cycle

The system by which years are marked historically in China was by the stem-branch or sexagenary cycle. This system is based on two forms of counting: a cycle of 10 *Heavenly Stems* and a cycle of 12 *Earthly Branches*. Each year is named by a pair of one stem and one branch called a Stem-Branch (*gānzhī*). The Heavenly Stems are associated with *Yin Yang* and the *Five Elements*. Recent 10-year periods began in 1984, 1994, and 2004. The Earthly Branches are associated with the 12 signs of the zodiac. Each Earthly Branch is also associated with an animal, collectively known as the *Twelve Animals*. Recent 12-year periods began in 1984, 1996 and 2008.

Within the Heavenly Stems system the year is advanced up by one per year, cycling back to year one after the last (year ten). Similarly the Earthly Branches also advances by one per year, cyclically. Since the numbers 10 (Heavenly Stems) and 12 (Earthly Branches) have a common factor of 2, only 1/2 of the 120 possible stem-branch combinations actually occur. The resulting 60-year (or sexagesimal) cycle takes the name *jiǎzǐ* after the first year in the cycle, being the Heavenly Stem of *jiǎ* and Earthly Branch of *zǐ*. The term "jiǎzǐ" is used figuratively to mean "a full lifespan"—one who has lived more than a *jiǎzǐ* is obviously blessed. [2]

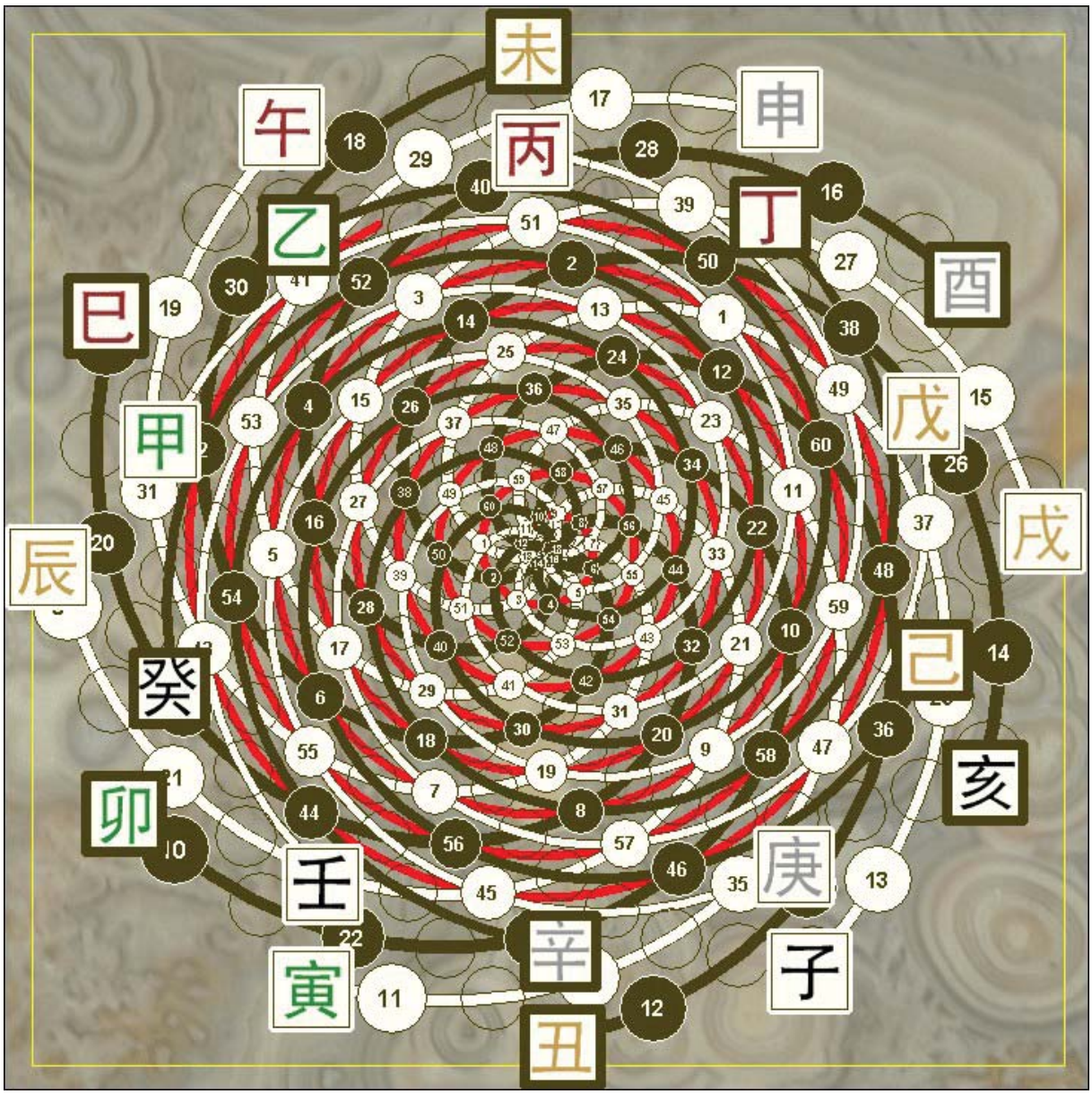

Fig. 2 Phyllotactic representation of Chinese **calendar** *60-year cycle*

2.2 Maya calendar 260-year cycle

Tzolkin is the name bestowed by Mayanist scholars upon the version of the 260-day Mesoamerican calendar which was used by the Maya civilization of pre-Columbian Mesoamerica.

The word, meaning "count of days", was coined based on Yukatek Maya.

The Maya used several cycles of days, of which the two most important were the Tzolkin, or Sacred Round of 260 days and the approximate solar year of 365 days or Haab.

In the 260-day cycle 20 day names pairs with 13 day numbers, totalling a cycle of 260 days. This cycle was used for divination purposes, it foretold lucky and unlucky days. The date of birth was also used to give names to both humans and gods in many Mesoamerican cultures, some cultures used only the calendar name whereas others combined it with a given name. Each day sign was presided over by a god and many had associations with specific natural phenomena. [3]

Fig. 3 The Aztec (also known as Mexica or Nahua) calendar is derived from the Maya

The cycle of twenty day names should really be thought of as an unending circle, or one eternal round. That is how it is depicted on the famous Aztec Calendar stone. The twenty glyphs are shown in the outer circle, beginning at the top and proceeding counterclockwise. One reason that the circular form is important is that it is often important which figures are opposite each other because the often form opposing pairs. For example, the serpent is opposite from the eagle.

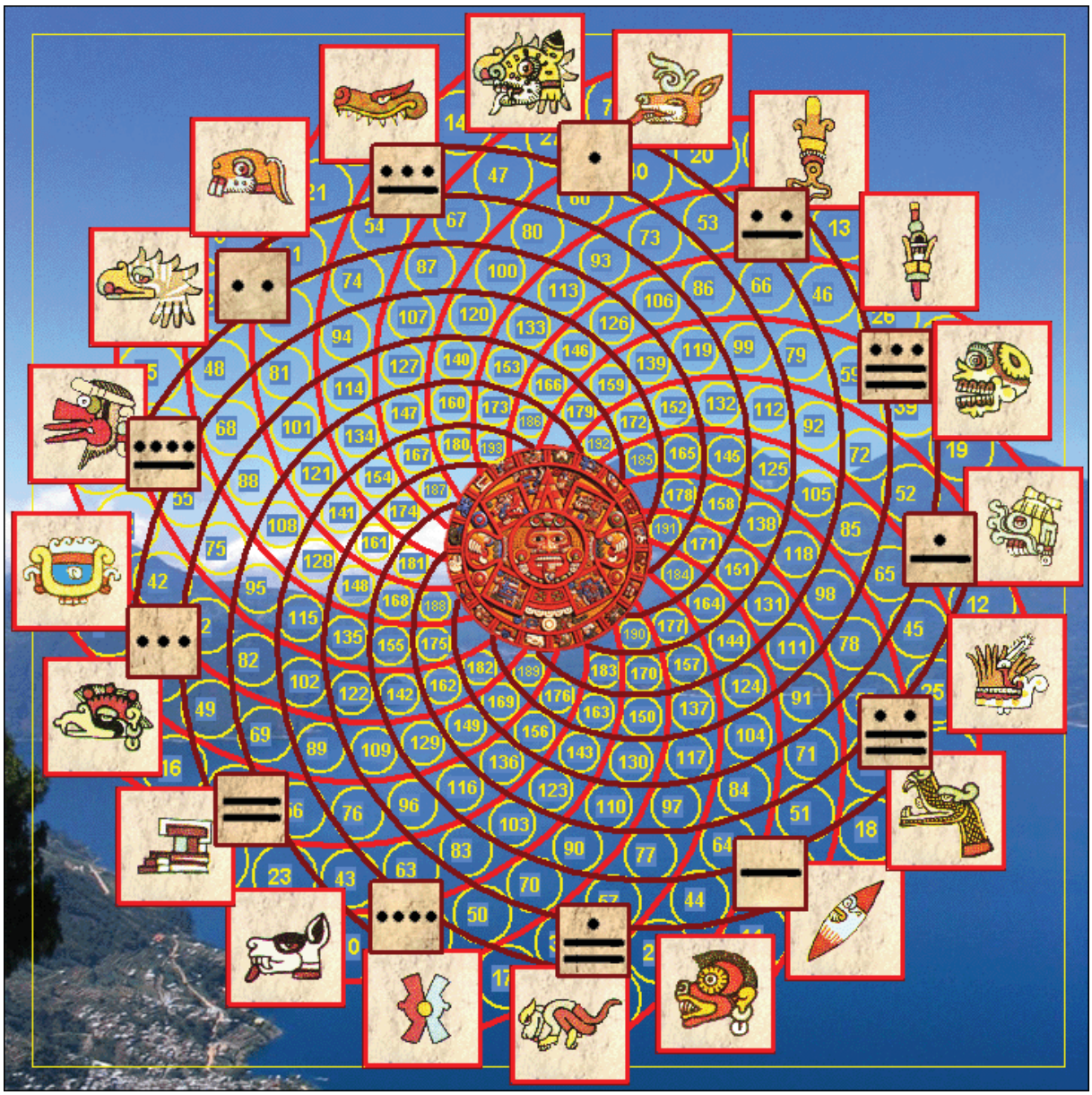

Fig. 4 Phyllotactic representation of Tzolkin. The relative positioning of gliphs in a circle of the new calendar dial is broken, but the opposition in pairs is kept.

2.3 Daaríisky Krugolét Chislobóga 144-year cycle

Krug = a circle; lét = years. Krug-o-lét = circle of years

Chislo = a number; Bog = God

An ancient Slavic calendar has been cancelled by tsar Peter the Great at 1700. Someone consider that there were the calendar “Daaríisky Krugolét Chislobóga”

which was based on modules 9 and 16. So, the cycle of years consisted of 9x16=144 parts. [4]

Fig. 5 Phyllotactic representation of Daaríisky Krugolét Chislobóga. Phyllotactic pattern 9x16=144

2.4 Metonic 19-year lunar cycle

In astronomy and calendar studies, the *Metonic cycle* or *Enneadecaeteris* (from Greek words for nineteen years) is a period of very close to 19 years which is remarkable for being very nearly a common multiple of the solar year and the synodic (lunar) month. The Greek astronomer Meton of Athens observed that a period of 19 years is almost exactly equal to 235 synodic months, and rounded to full

days counts 6940 days. The difference between the two periods (of 19 years and 235 synodic months) is only a few hours, depending on the definition of the year.

To keep a 12-month lunar year in pace with the solar year, an intercalary 13th month would have to be added on seven occasions during the nineteen-year period. Meton introduced the cycle in circa 432 BC but it was actually known earlier by Babylonian astronomers. [5]

2.5 Scandinavian lunar calendar

The *Runic calendar* is a perpetual calendar based on the 19 year long Metonic cycle of the Moon.

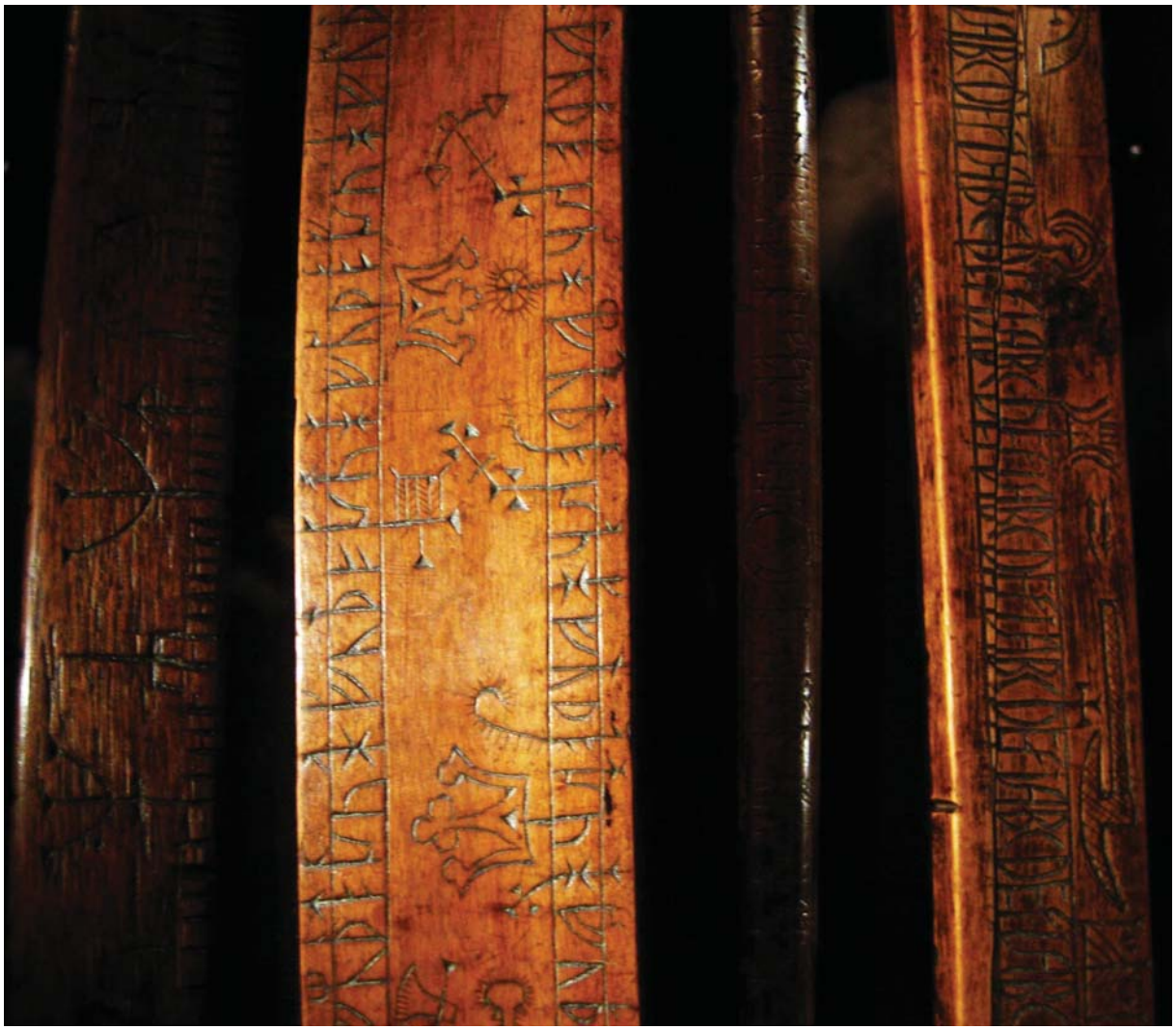

Fig. 6 Rune staffs at the Museum of History in Lund, Sweden.

Also known as a *Rune staff* or *Runic Almanac*, it appears to have been a medieval Swedish invention. Runic calendars were written on parchment or carved onto staves of wood, bone, or horn. The oldest one known, and the only one from the Middle Ages, is the Nyköping staff, believed to date from the 13th century. Most of the several thousand which survive are wooden calendars dating from the 16th and the 17th centuries. [6]

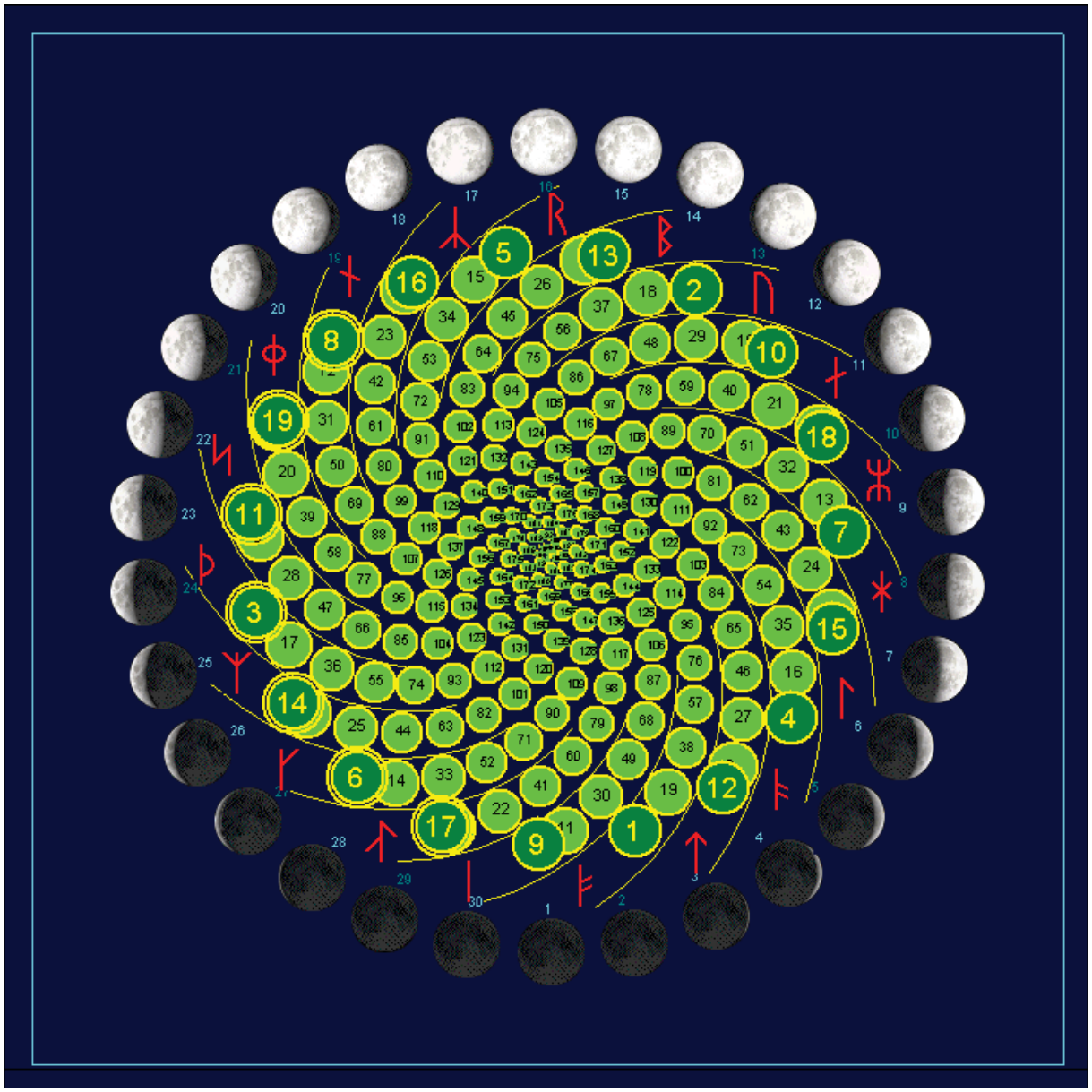

Fig. 7 Phyllotactic pattern of Metonic cycle and Scandinavian lunar calendar. The epacts are located under the order. The years with the embolismic or intercalary month are located in one sector (marked by the numbers 8, 19, 11, 3, 14, 6, 17 in dark-green circles on the periphery). Runes of Scandinavian lunar calendar are designated by red.

2.6 Perpetual calendar with possibility of definition of Easter day

Easter is a moveable feast, meaning it is not fixed in relation to the civil calendar. Eastern Christianity bases its calculations on the Julian Calendar whose March 21 corresponds, during the 21st century, to the 3rd of April in the Gregorian Calendar, in which calendar their celebration of Easter therefore varies between April 4 and May 8.

The date for Easter is determined on a lunisolar calendar. The First Council of Nicaea (325) established the date of Easter as the first Sunday after the full moon (the Paschal Full Moon) following the northern hemisphere's vernal equinox. [7]

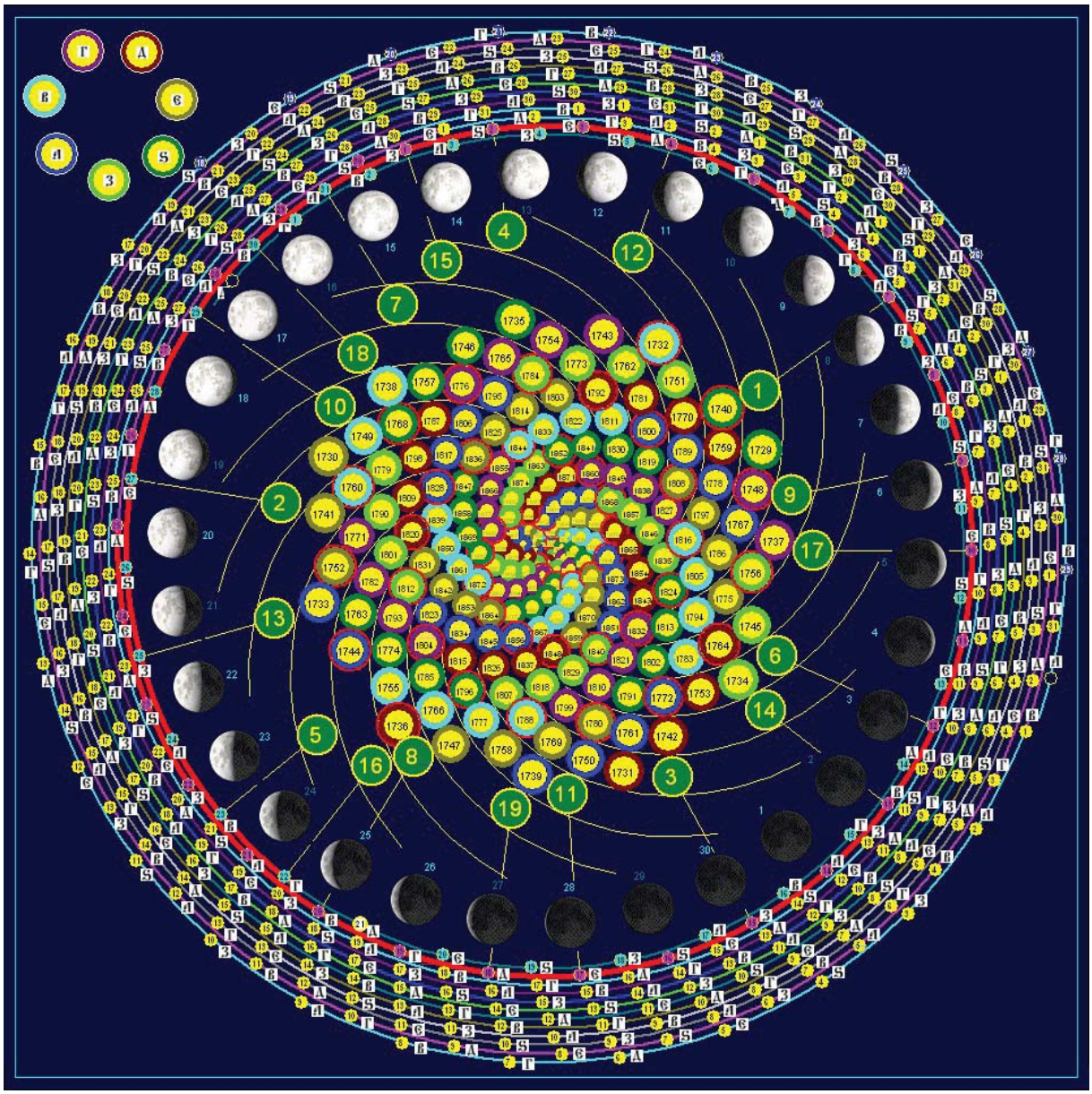

Fig. 8 Phyllotactic pattern of Perpetual calendar with possibility of definition of Easter day.

The calendar makes it possible to identify the day of the week and an Easter day of any year. Accounts and designations are made according to the Julian calendar.

The years are placed in the centre of the picture according to Metonic Cycle. The days of one year are located on the external spiral. The months of the year are noted by different colours of this external spiral.

The period when Easter is possible (March 22 – April 25) is marked by a red thick line.

We should know the *vrutseleto letter* to identify the day of the week. The correspondence of a year to a vrutseleto letter is marked by some colour in the upper left corner of the picture. The vrutseleto letters are depicted on the white background.

In order to determine the Easter day celebration we should move from the centre to periphery on the following rout: a chosen year (we find out and remember the vrutseleto letter) – the spiral of inner inflorescence – Golden number (1-19) in the green circle – a line space in radial direction to a date situated on the external spiral – along this external spiral clockwise to the nearest vrutseleto letter, which we remembered. A number near letter will show the date of Easter celebration according to Julian calendar.

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5. **http://en.wikipedia.org/wiki/Metonic_cycle**

6. **http://en.wikipedia.org/wiki/Runic_calendar**

7. **http://en.wikipedia.org/wiki/Easter**