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PERIODIC 3-D NETWORKS AND SPONGE SURFACES AND THE ASSOCIATED SPACE PHENOMENOLOGY. MORPHOLOGICAL FUNDAMENTALS OF 3-D SPACE THEORY.

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Abstract:

'Morphologically speaking', at the care of the **creative process of our habitat development and its architectural design and articulation** is the **manipulation of space** and its prime associated 3D space features of **network** structures, space-subdividing **partitions** and **cell-packing** agglomerations.

1. Networks may represent the structure of almost any abstract or physical plurality that may exist, in the world of phenomena of the biological-physical-material-spiritual domains, on every possible scale, from the Nano-molecular to the cosmological..
2. Surfaces-partitions, mostly sponge-like (hyperbolical) space sub-divisions, are probably the most abundant forms in nature, on every possible scale of physical-biological reality.
3. Cellular space close packings, and their polyhedral definitions, represent the morphological essence of the habitat solutions of living multitudinous societies in zoology, botany or the virus domains and the structures of all material crystalline aggregations as well.

Periodicity of 3-D space forms features the negation of the total formal freedom (arbitrariness), and that by imposing topological parametral similarities and symmetry constraints. The tension between the free form and the topologically-symmetrically constrained periodic configurations are at the heart of the design art.

The 3-D Space Phenomenology – THE QUINTET ENSEMBLE

- Every 3-D network is associated with a dual, its uniquely determined reciprocal network. Networks in general, and periodic networks in particular, come in dual pairs.
- Every 3-D network is genetically associated with a close packing solid (or solids) and therefore, any dual networks pair leads to two modes of close packing cellular-polyhedral solutions.
- Any pair of dual networks is associated with a unique, topologically determined sponge surface, subdividing the space between the two.

If the dual networks are periodic in nature to a point of adhering to a certain specific symmetry group, the two associated cellular packing modes and the subdividing sponge surface relate to same symmetry regime.
The described five features of the 3-D space: the dual networks pair, the two associated packing modes (and their representative packing solids) and the associated space partition, all together, represents the **'Quintet Ensemble'** which encompasses the essence of the 3-D space phenomenology.
The number of 'Quintet Ensembles' in 3-D space is infinite, implying that the number of dual network pairs (and networks in general) and the number of sponge surfaces subdividing 3-D space into two complementary spaces are stretching to infinity as well.

The emergent fundamental morphological principles of 3-D space theory may be formulated as follows:

1. Every four components of the 'Quintet Ensemble' can be accurately defined and derived from the fifth.
2. The connectivity values of the dual networks pair are the same and equal to the genus value of the associated surface partition, subdividing the space between the two. Con.(net) = g (surface).
3. Every continuous 3-D network is embeddable in an unbounded continuous 3-D sponge surface (2d-manifold), thus featuring a specific '**unihedron'**, adhering to specific Val., Σα&g prime parameters, where $\mathbf{\Sigma\alpha_{av.} = 2\pi(Val. - 1).}$

The presentation will be saturated with illustrations and will expand on the above 3-D networks, the related close space packing cellular agglomerations and the associated sponge surface space subdividing partitions which represent the most important morphological features of our architectural design imagery and principal, visually embraced, research tools of 3-D space phenomenology.
The presentation deals with their binding (theoretical) relations which could eventually contribute to the evolution of 3-D networks theory.

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The Morphologically Associated Quintuplet Phenomena of 3D Space

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Abstract
3D space phenomenology of our habitat is mostly concerned with the morphological features of network structures, cellular polyhedral close packing agglomerations and space subdividing partition surfaces.
They form the core of our imagery of the physical and the virtual-imaginary space we live in. Their manipulation determines the structure of our habitat, provides for its architectural design and consequently for its formal evolution and development.
The number of topologically different space networks, sponge surface partitions and cellular space-packings amounts to infinity, even when topologically-symmetrically constrained, as periodic features.
Observing the field of 3D space phenomena it transpires:

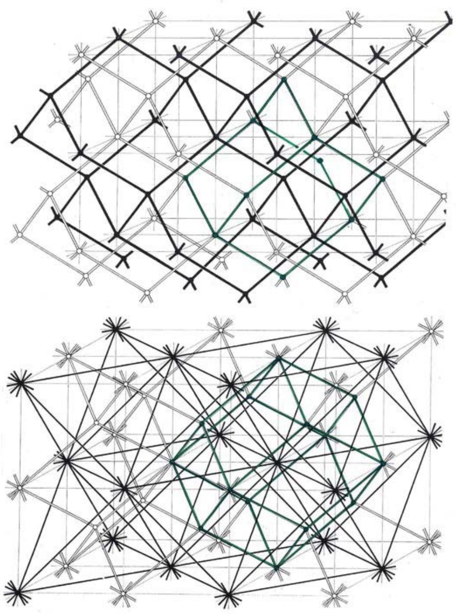
- Networks, in general, come in dual reciprocally related pairs.
- Every 3D network is genetically associated with a cellular close packing of polyhedral (mostly finite) volumetric solids.
- Any pair of dual networks is associated with a unique, topologically and symmetrically determined hyperbolical sponge-surface, subdividing the entire space between the two.

It transpires that every dual networks pair, the associated sponge surface, subdividing between the two and the two associated close-packing modes describe an inter-relating **quintuplet,** in which **every four components can be accurately defined and derived from the fifth.**
The quintuplet phenomena are at the core, and represent the essence of our 3D space phenomenology, and consequently must play a significant role in the evolving networks theory.

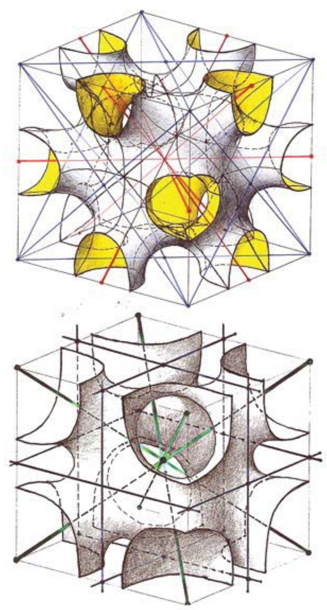
Introduction

The 3D space phenomenology of our habitat is mostly concerned with the morphological features of network structures, cellular polyhedral close packing agglomerations and space subdividing partition surfaces.
They form the core of our imagery of the physical and the virtual-imaginary space we live in. Their manipulation determines the structure of our habitat, provides for its architectural design and consequently for its formal evolution and development.

1. **Networks**, a connected assembly of vertices and edges, may represent the structure of almost any abstract or physical plurality that may exist, in the world of phenomena of the biological-physical-material-spiritual domains, on every possible scale, from the nano-molecular to the cosmological. They are the morphological essence of our built structures of products, buildings, urban sprawls, regional fabric and inter-national boundaries and all the associated transportation-communication interaction systems and installations of our living environment space.

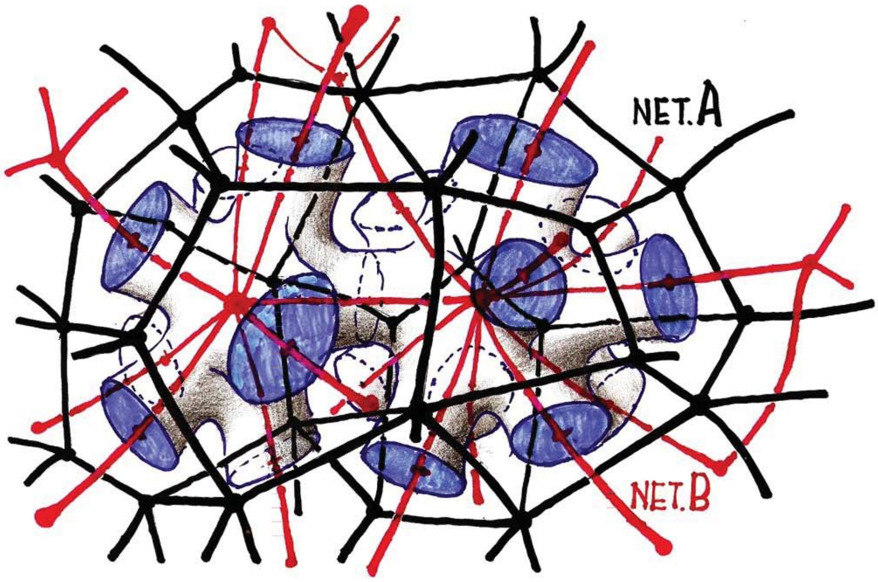


UNIFORM TETRAVALENT 'DIAMOND' LATTICE AND UNIFORM DODECAVALENT 'OCTET' LATTICE AND DUALS.

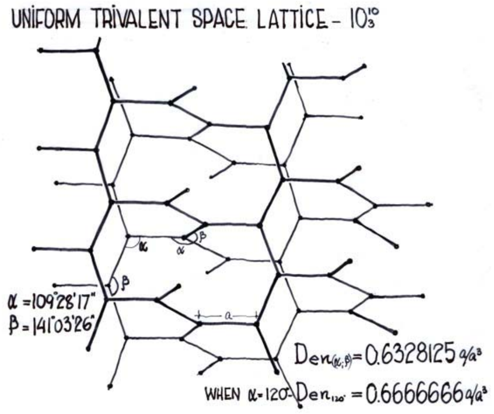


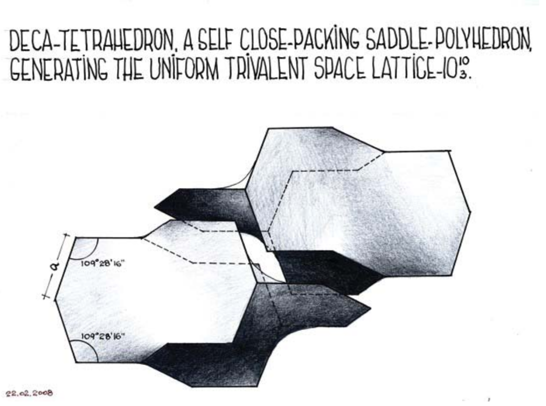
SPONGE SURFACES SUBDIVIDING SPACE BETWEEN TWO DUAL NETS

2. **Surface-partitions**, plane, spherical but mostly hyperbolical, sponge-like space sub-divisions, are probably the most abundant forms in nature, on every possible scale of the physical-biological reality. Partitions define our personal, family or communal and national territorial-spatial expansion boundaries and the limits of our control, thus defining the boundaries between the **interior** and the **exterior** as predominant features of our environment. [1], [2], [3].

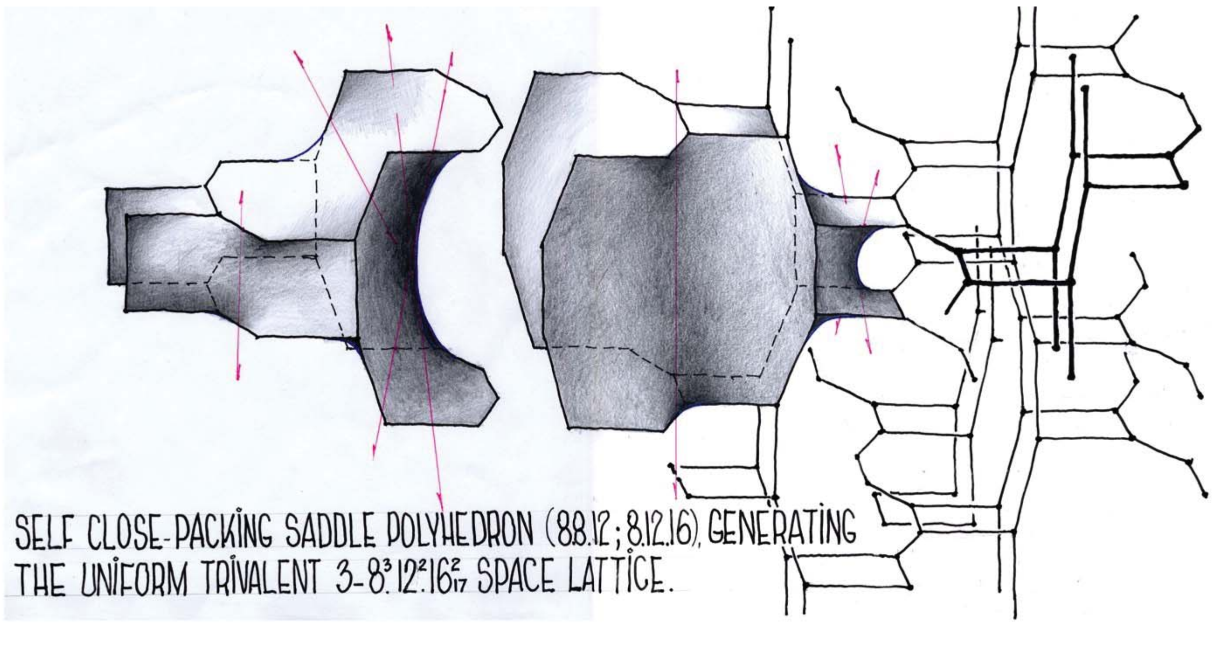


TWO (NON PERIODIC) DUAL NETWORKS, A & B AND THE SPONGE-SURFACE, SUBDIVIDING SPACE BETWEEN THE TWO.





3. **Cellular**, loose or compact close **space-packings** and their polyhedral entities-solids, represent the morphological imagery of the segregated habitat solutions of living multitudinous societies in zoology, botany or the virus domains and the structures of all material crystalline aggregations as well. [4], [5], [6].



The finite cell units, mostly shaped like “saddle polyhedra”, (having hyperbolical curved faces, with one or two, and no more than two faces, meeting at every edge) conform with the Euler's theorem and formula of V-E+F=2K (where V,E,F&K stand for vertices, Edges, Faces and the Euler characteristic K, respectively). For finite polyhedra K=2.

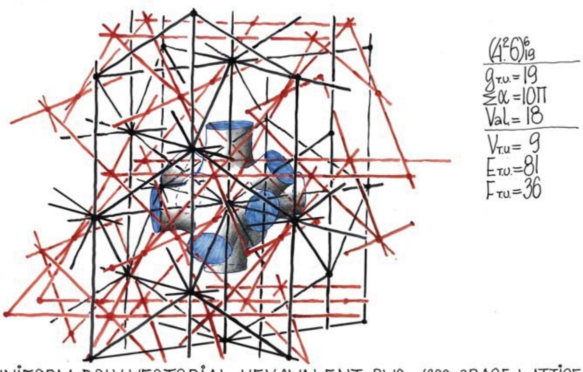
The number of topologically different space networks, partition-surfaces and cellular space-packings is infinite (for each category), even when periodic in nature, due to topological similarities' or symmetry constraints. [7], [8], [9].

The interplay and the resulting tension between the completely free (arbitrary-chaotic) form and topologically-symmetrically constrained periodic configurations are at the heart of the design art and its phenomenology.

The Primary Infer-Related (reciprocal) 3D Space Quintuplet Phenomena

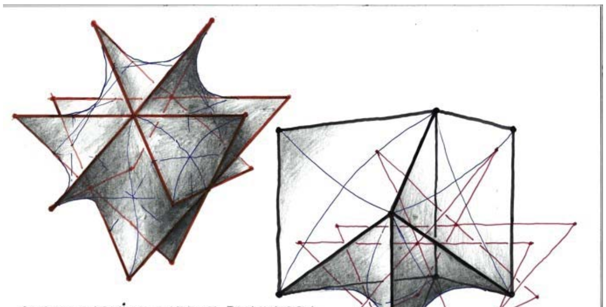
The topological constraints of continuously connected (unbounded) networks in 3D space dictate the following:

- Every 3d network is associated with a dual, its uniquely determined reciprocal network, being uniquely derived from the first.
  Networks in general and periodic networks in particular, **come in dual pairs.**
- Every 3D network is genetically associated with a cellular close-packing of polyhedral (mostly finite) volumetric solids, and therefore, any dual networks pair leads to two modes of polyhedral-cellular close-packing solutions.
- Any pair of dual networks is associated with a unique, topologically determined hyperbolical sponge-surface, subdividing the space between the two. Every such unbounded hyperbolical sponge surface, whether periodic-uniform or hopelessly chaotic, subdivides the entire space between two complementary labyrinthine spaces, with two tunnel systems, the axes of which feature two interwoven dual space networks.
- To construct, from a given network A, its dual network B, the following method should be employed:
  1. Perceiving network A as its associated cellular close-packing agglomeration of polyhedral solid cells (with V_A; E_A & F_A).
  2. Geometrically determining the centroids of all the polyhedral volumes of A.
  3. For network B to emerge, we have to join all the previously determined centroids with edges in such a way that every edge is passing through a polyhedral face of A, with the resulting relations: $E_A \equiv F_B$; $E_B \equiv F_A$; and therefore, the edge valency of any vertex of A equals the number of faces of the corresponding unit cell B and vice versa.
     It transpires that the number of different packing solids of 'A' corresponds to same number of different vertex-figures of 'B', and vice versa. If a network is periodic in nature to a point of adhering to a specific symmetry group, its dual network, the associated cellular packing configurations and the subdividing sponge surface, relate to same symmetry regime.
     The described five features of 3D-Space, namely the **dual networks pair,** the **two associated close-packing modes (**with their respective polyhedral solids and the associated **hyperbolical sponge surface,** all together represent a **'quintuplet assembly'** which encompasses the essence of the 3D space phenomenology. The number of such 'quintuplets' in 3D space is amounting to infinity, implying that the number of dual network pairs (and networks in general), the number of sponge-surfaces subdividing between the two and the number of close packing cellular solutions are stretching to infinity as well.

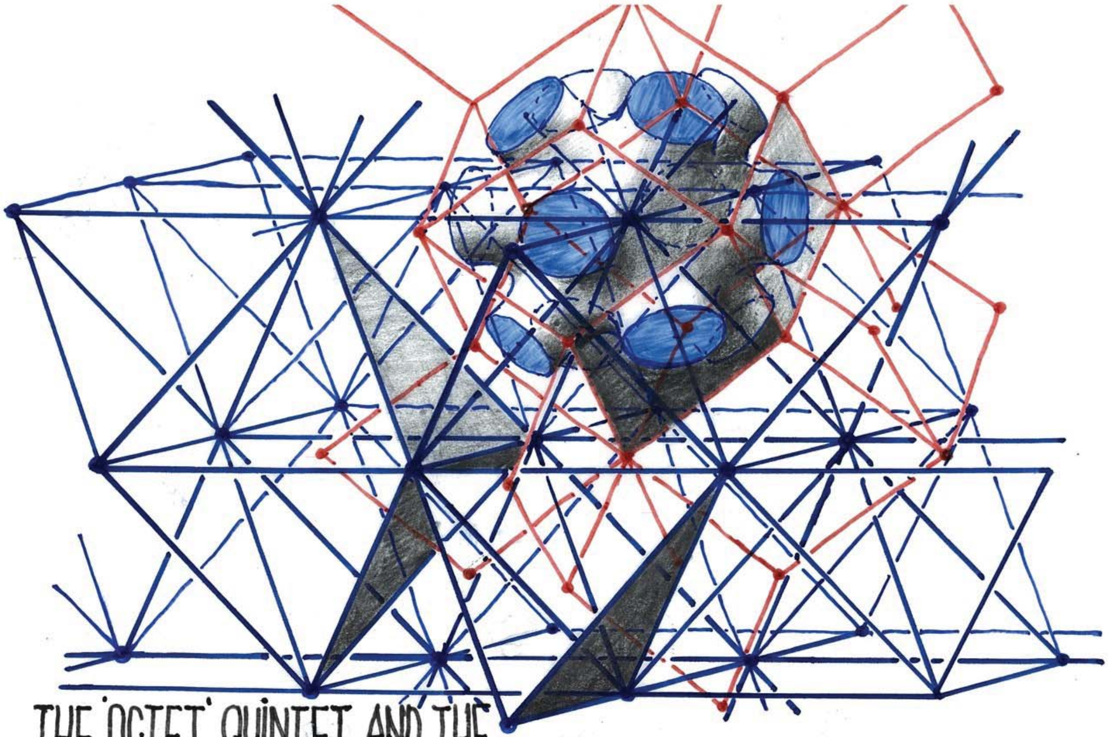


UNIFORM POLY-VECTORIAL HEXAVALENT $PV.6-4^3.6^9_{19}$ SPACE LATTICE,
AND IT'S DUAL (NON-UNIFORM) DECAVALENT $PV.10-4^{28}_{19}$ SPACE LATTICE.

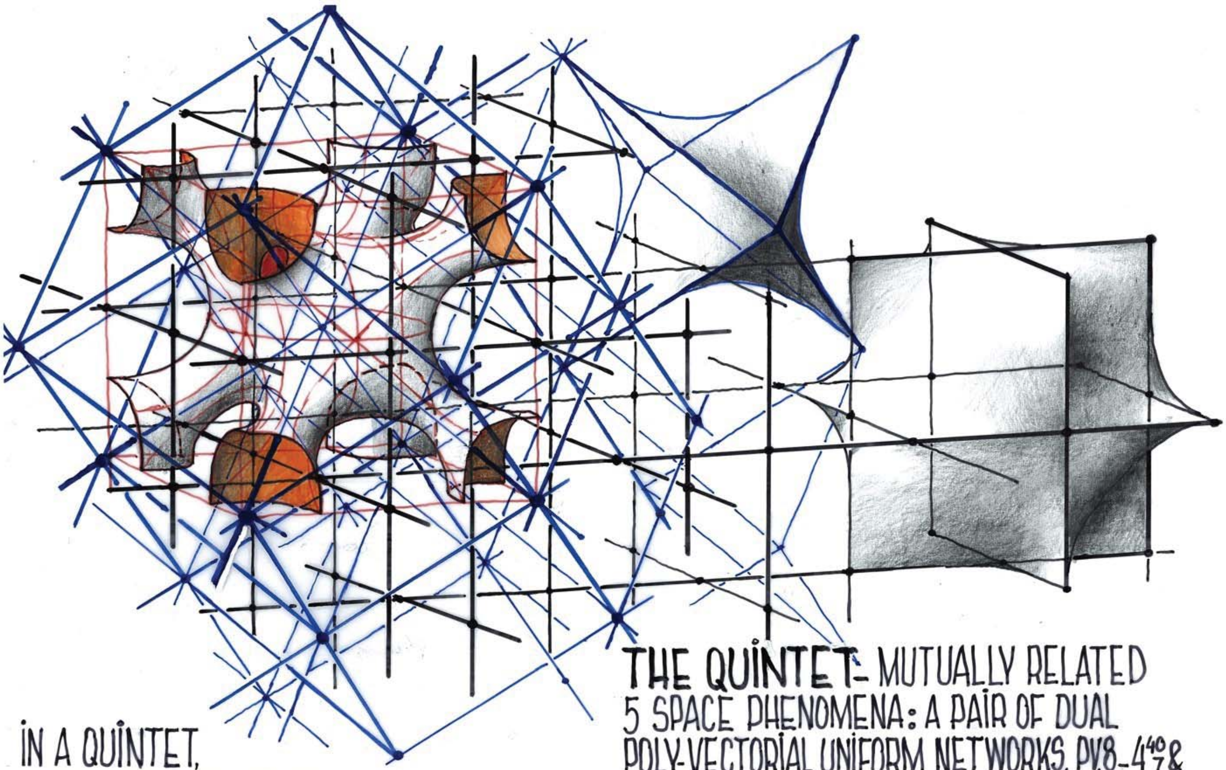
The emergent fundamental morphological principles of 3D-space theory may start with the following formulation:



CLOSE PACKING SADDLE POLYHEDRA,
DECAHEDRON GENERATING THE
UNIFORM $PV.6-4^3.6^9_{19}$ SPACE LATTICE AND THE
HEXAHEDRON GENERATING IT'S DUAL $PV.10-4^{28}_{19}$.



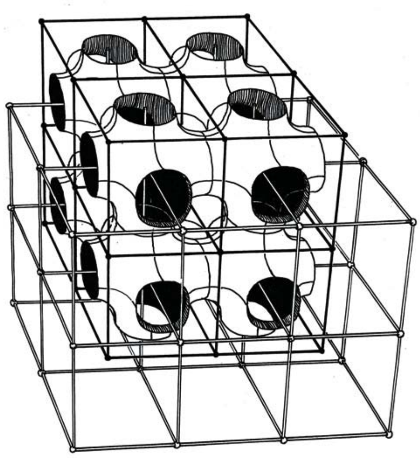
THE 'OCTET' QUINTET AND THE
ASSOCIATED DUAL NETWORK, THE SPONGE PARTITION AND PACKING SOLIDS.

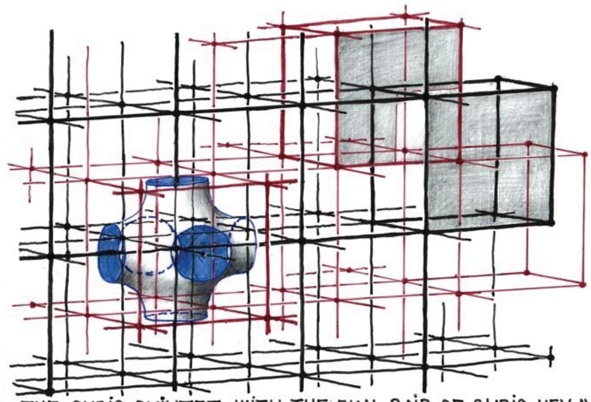


IN A QUINTET,
EVERY FOUR SPACE FEATURES ARE
RECIPROCALLY DERIVED FROM THE FIFTH.

THE QUINTET- MUTUALLY RELATED
5 SPACE PHENOMENA: A PAIR OF DUAL
POLY-VECTORIAL UNIFORM NETWORKS, $PV.8\text{-}4^{40}_{7}$ &
$PV.4\text{-}6^{80}_{\cdot}8^{6}_{7}$, THE SPONGE SURFACE PARTITION
AND THE TWO CLOSE-PACKING SOLID CELLS.

1. Every dual networks pair, the associated sponge surface subdividing between the two and the two associated close-packing modes describe an inter-relating ‘quintuplet in which **every four components can be accurately defined and derived from the fifth.**
2. The average connectivity values of the dual networks pair are the same and equal to the average genus value of the associated partition surface, subdividing the space between the two.
3. If the ‘quintuplet is periodic-symmetrical in nature, all the five associated components share in the same symmetry regime (adhering to same symmetry group).
4. The particular inter-relations between any two dual networks may be reflected through existing relations between their close-packing modes:
   $\mathbf{V_A \equiv C_B}$; $\mathbf{E_A \equiv F_B}$; $\mathbf{F_A \equiv E_B}$; $\mathbf{C_A \equiv V_B}$, with V_A; E_A; F_A&C_A (and same for V_B ;E_B;F_B;& C_B) corresponding to vertices, Edges, Faces and Cell units of ‘A’, respectively.
5. If the two dual networks are identical (as with the Cubic or the Diamond lattices) the surface partition subdivides the entire space into **two identical complementary sub-spaces**, thus giving rise to a third (highly periodic) lattice, composed entirely of 2-fold rotation axes, each of which is rotating network ‘A’ into its dual ‘B’. All such rotation axes are embedded in the sponge surface partition and represent its tessellation tiling, composed of identical surface unit. [1]





THE CUBIC QUINTET, WITH THE DUAL PAIR OF CUBIC HEXAVALENT PV.6-$4^{12}.6^{8}_{3}$ SPACE LATTICES, SPONGE PARTITION AND THE PACKING CELLS.

6. The number of identical network pairs and the associated sponge surfaces that subdivide 3D space into two identical sub-spaces is amounting to infinity.

In conclusion

In his monumental publication: **"Structural Inorganic Chemistry"** (1962), in a chapter discussing the **'Geometric Basis of Crystal Chemistry',** referring to 3D networks, A.F.Wells makes a startling factual observation: **"The theory of these nets does not appear to be known,** and in fact no attempt to derive them systematically seems to have been made"..(pp. 101). Even his efforts ('Three Dimensional Nets and Polyhedra'-1977) did not help to resolve the issue in a meaningful way. [10], [11], [12].

A comprehensive theory in any research domain may emerge only after the domain's phenomenology is accounted for and comprehended.

It is plausible to hypothesize that if such a theory would have been in existence some decades ago, quasi-crystalinity would have made its appearance much earlier.[13].

Many aspects relating to network's phenomenology are still inviting research. Such a research program, part of the authors agenda through the last years, may include the following.

- Networks topological-symmetrical nature, variability and constraints and the observed impact on their imagery.
- Their exhaustive enumeration, categorization and classification.
- Their primary form-shaping parameters and their mathematical interplay.
- Their inter-relation with space subdividing surfaces and close (compact) packing of space.
- Enquiry into their density properties and evolution, and their stability as physical structures, and many more. Gaining insights into these aspects must be a prelude to a comprehensive theory formulation of networks in 3D space.[14].

The presented **'quintuplet of the associated 3D space phenomena'** are central to our perception and understanding of 3D space in general and that of our habitat environment in particular. 3D networks and the associated hyperbolical partition surfaces and the twin close-packing modes

represent the most important morphological features of our architectural design imagery and primary, visually embraced notions and features of our 3D space phenomenology.
The presentation deals with their binding inter-relations which could eventually contribute to the evolution of 3D networks theory.

Glossary

Polyhedral Envelopes

Polyhedra – **tessellations** of **unbounded (2d) surfaces,** are composed of vertices (V), edges (E) and faces (F), with two faces only meeting in every edge. Polyhedra may be **spherical, toroidal** or **hyperbolic (sponge polyhedra).** Their **primary parameters** are: **Valency (val.)** –number of edges meeting in a vertex; - **Sum of angles** in a vertex, or (when the faces are not planar) the **total surface curvature** of a **vertex region; Genus (g)-** the maximal closed circuit cuts of the surface which leave it un-separated into two (dis-attached) parts.

Space Networks

A network constitutes an array of vertices and inter-connected line segments – edges. Shortest closed circuits of edges may describe **polygons** which in themselves constitute faces of **polyhedral unit cells.** Aggregation of such polyhedral cells, in its totality, describes a close **compact space packing** resulting in the network structure. **Uniform Polyhedra** and **Uniform Networks** result from the imposition of **symmetry constraints,** enforced by a specific **symmetry space group** and are characterized by **identical vertex figures,** meaning that the environment observed from every vertex is one and the same.
The defining symmetry space group implies the existence of repeating **symmetry space units** either as **Elementary Periodic Regions (E.P.R)** or as a **Translation Unit (T.U.)** characteristic of the space network, which include complete representation of the topological-symmetrical features of the configuration.
Network Duality: When the centroids of the close – packed polyhedral cells of a given A network are inter connected, network B, it's dual, is generated, with one edge of B penetrating each of A faces (and vice versa).
Between any pair of dual networks a partition may be drawn in the form of a sponge polyhedron or a curved, mathematically determined sponge surface.
The dual networks and the associated sponge partition share same symmetry characteristics and the **connectivity** of the networks pair and the **genus** of the partition are of the same value.

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