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Paper : **ADVANCED GEOMETRY OF MODULAR TILES**

Topic: Architecture and Mathematics

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References:
[1] Hauer, E.: Architectural Screens and Walls, Princeton Arch. Press, New York, 2004
[2] Moussavi, F.: The function of ornament, Actar, Barcelona, 2006
[3] Shubnikov, A.V., Koptsik V.A.: Symmetry in Science and Art, Plenum Press, London – New York, 1977

Abstract:

The aim of this paper is discussion of symmetry groups of ornaments, function of ornament with regard to the digital fabrication, application of theoretical knowledge in generation of three dimensional parametrical ornamental structures and presentation of achieved results.

Based on the seventeen wall paper groups we try to extend two-dimensional ornament into digital three-dimensional ornamental pattern using appropriate mathematical and geometrical rules. The mathematical basis of our construction is the theory of simple and multiple antisymmetry. A large collection of different ornamental patterns can be obtained by using only six modular prototiles. E.g., beginning from the symmetry group *pmm* and using 2-multiple antisymmetry, we obtain 840 different patterns.

We established the digital flow from design to fabrication using 3d NURBS modeling, visual programming software for code generation and we fabricated our 3D ornamental pattern with robotic arm. This approach enabled huge flexibility in design and real time feedback to the designer concerning: size of robot arm, availability to all desired positions regarding to the chosen tool and robot axes restriction. In the same time our approach enabled to explore how digital prefabrication can contribute to conceptual exploration and how new technology can influence existing design.

The group of the students had a possibility within the course "Design of specialized topics" to use our digital flow and at the end of our work we will present their results.

Images of prefabricated 3d ornamental patterns

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Keywords:
parametric design, symmetry of ornamental group, antisymmetry, modular patterns, robotic arm, digital fabrication

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Abstract:

The aim of this paper is to discuss ornamental symmetry groups, the function of ornaments with regard to digital fabrication, the application of theoretical knowledge in the generation of three-dimensional parametric ornamental structures and the presentation of achieved results.

Based on the seventeen wallpaper groups, we try to extend a two-dimensional ornament into a digital three-dimensional ornamental pattern using appropriate mathematical and geometrical rules. The mathematical basis of our construction is the theory of simple and multiple antisymmetry. A large collection of different ornamental patterns can be obtained by using only six modular prototiles. E.g., beginning with symmetry group *pmm* and using 2-multiple antisymmetry, we may obtain 840 different patterns.

We have established a digital flow from design to fabrication using 3d NURBS modeling and visual programming software for code generation, and we have fabricated our 3D ornamental pattern with the robotic arm. This approach has proved to enable huge flexibility in design and real-time feedback to the designer concerning: size of the robot arm, availability of all desired positions regarding the chosen tool and robot axes restriction. At the same time, our approach has allowed us to explore how digital prefabrication can contribute to conceptual exploration and how new technology can influence existing design.

A group of students had a possibility to use our digital flow within the course “Design of specialized topics” and the results of their work are presented later in this paper.

Keywords: parametric design, symmetry of ornamental group, antisymmetry, modular patterns, robotic arm, digital fabrication

1. Introduction

The possibility to use digital tools to generate 3d models and realize architectural building elements has led, among other things, to the re-establishment of ornaments in architectural practice. The complexity of the subject of ornamentation as it occurs in art, mathematics, geometry, architecture, religion and psychology has inspired architects and designers to reconsider this field again and again and has also represented inexhaustible motivation for new design. The re-establishment of ornaments in architectural practice raises the issue of how ornaments may be given new quality and new use value through the use of contemporary materials and fabrication processes. Is it possible to apply the potential and properties of modularity which the ornament possesses in the age of non-standard architectural design, while avoiding an easily recognizable repetition of basic geometric shapes? In what way is it possible to transform mathematical and geometric laws of 2d ornamental wallpaper elements into parametric 3d ornaments? What constraints must exist in 3d geometry if a specific method is selected for the fabrication of elements using the robotic arm? How may individual digital design be transformed into digital robotic code? What limitations exist in design with respect to the capacities of the robot?

As part of the course “Design of specialized topics” on the subject of Computer Serendipity – The Robot as the Architect’s Best Friend, held during the summer semester 2011 at the Faculty of Architecture in Graz, the students tried to produce design and engineering solutions to solve the above problems.

2. Why ornaments?

Through the history of architecture the role and denotation of ornament was shaped by cultural, intellectual and technical development. The decreasing and increasing use of ornaments in architecture was linked to their use as superficial, mostly two-dimensional and symmetrical elements on the façade. The development from Speiser`s exploration of ornament as a matter of symmetry to Shubnikov`s analysis of the symmetry method for revealing the invariants of transformation to Semper´s theory of ornament and Loos`s opposition to it, to Moussavi`s classification of the ornament based on depth material or effect indicates the complexity of the different approaches to the subject-matter of ornamentation.

The continued use and development of ornaments indicates there is a specific psychological base for using ornamentation. In psychology the aesthetic value of ornaments is related to symmetry. Experimental psychology has proved that people can recognize symmetric forms in less than a twentieth of a second. The eye is very fast in the detection of vertical symmetry. Locher and Nadine have proved that after the recognition of symmetry the eye starts seeking only the superfluous elements of the composition, while the other part of composition is accepted. Symmetry is further related to our perception and perception is related to our sense of beauty. According to Hekkert , design is considered beautiful or pleasing when a great effect is attained with a minimum of means and when our senses perceive this hidden structure.

Cognitive psychology and Gestalt psychology can explain our aesthetic reaction to ornaments. The brain tries to make a group of elements and to find a law of composition. According to Shubnikov , the aesthetic effects resulting from the symmetry (or other laws of composition) of an object lie in the psychic process associated with the discovery of its laws. According to Leder , most people consider aesthetic whatever is plausible.

The beauty of ornamental pattern can be found in the rhythmical repetition of motifs with conspicuous dominants and a distinctively emphasized arsis. Ornaments underlie the architectural effects of buildings and are also vital for effects in the urban landscape . Developed in all historical epochs and in all cultural areas, the ornament was always a unique manifestation of figurative experience in many primitive cultures, performing not only decorative but also a pronounced magic and symbolic function. As the ornament developed, it went from natural motifs to stylization and further to strictly geometric shapes. It is often very hard to trace singular geometric motifs and to decide if we are dealing with an intrinsic stylization of figural motifs or if they are primarily conceived as an abstract form.

The aesthetic effect of ornaments on buildings has been explored in various ways in history. Ornaments on buildings were used in traditional society as an instrument of differentiation. The structural and functional requirements of a building, according to Semper , were subordinated to the semiotic and artistic goals of ornamentation. In the twentieth century with Modernism the ornament lost its social function and became unnecessary. For Loos , the modern society needs not emphasize individuality through buildings but on the contrary suppress it. Modernism tried by means of style to adjust changes in culture. The relationship between the interior and exterior of buildings changed. Modernism used transparency to replace ornament to achieve a conductive representation of architectural elements of space, structure and program. In this “transparent” paradigm the function of architecture was visible and readable in the urban setting.

In the 1970`s, Venturi and Brown formulated the critique of Modernism for the purpose of replacing transparency with décor. For them, décor gives a building a new meaning in the eyes of the public and helps to integrate it in the urban setting. Furthermore, Deconstructivism, as a development in postmodern architecture, uses the geometry of collage as a style instead of transparency and decor. The finished visual appearance of deconstructivist buildings is characterized by a stimulating unpredictability and controlled chaos. Whether an ornament is used as a contingent – matter of décor and communication, or a necessity like an effect or sensation – It is necessary and inseparable from the object.

Particularly new technologies in architecture have a huge influence on the further processing of the ornament as a non-standard element and new systems of production have opened up possibilities for their differentiation and customization. In contemporary architecture we can find examples of contemporary ornaments like laser-cut sheets (Christian Dior Ginza Store, Kumiko Inui), glass tubes (Louis Vuitton Roppongi Hills Store, Jun Aoki), perforated screens (Centre du Monde Arab, Jean Nouvel), color-coding effect (Laban Dance Center, Herzog & de Meuron) or silk screened images (Eberswalde Library, Herzog de Mouron, Thomas Ruff).

3. Geometry of wallpaper symmetry groups

From the aspect of mathematics and geometry, all periodic wallpaper ornaments may be classified in 17 groups . Each wallpaper group is characterized by specific plane transformations, which individual elements – the cells of ornaments – must satisfy to be classified in any specific group. The principal geometric property of a 2d cell is that it may be multiplied to cover a plane completely, through various specific transformations of the plane (translation, reflection, glide reflection and rotation). Namely, two congruent figures may be mirrored, using rotation or translation, whereas two symmetrical figures may be duplicated using reflection and glide reflection (reflection followed by translation along the direction of the reflection axis). When it comes to symmetry groups, there are only three different kinds of regular tessellation to be used to cover a plane completely. This type of tessellation is performed with the shapes of parallelepiped, equilateral triangle and regular hexagon. The unit cells of wallpaper ornaments are obtained when these basic geometric shapes are split into identical secondary elements. Figure 1 shows the plan elevations of basic shapes, cells generated from these basic shapes and the types of wallpaper ornament groups they constitute.

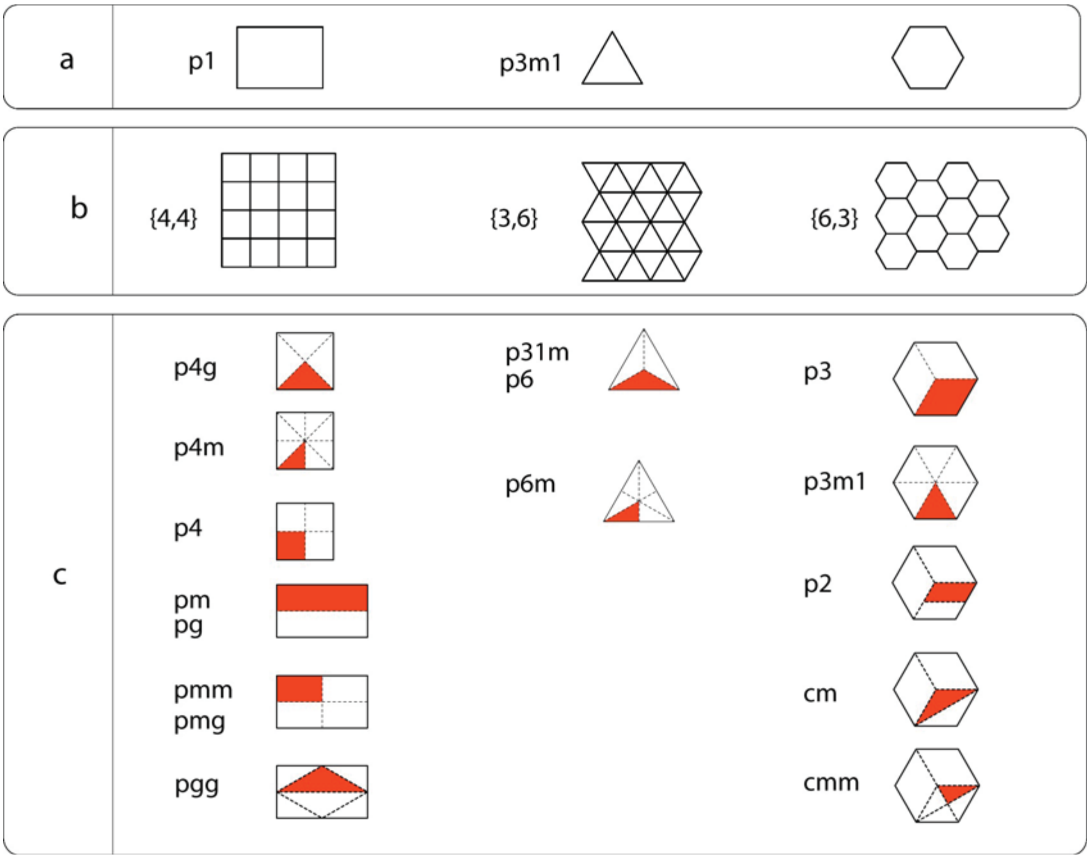

Figure 1: a. Basic shapes that may be used to cover a plane; b. Uniform tiling, plane covered with uniform geometry unit cells; c. Unit cells of specific wallpaper groups

3.1 Modularity

As the modularity will be considered the use of several basic elements (modules) for constructing a large collection of different possible (modular) structures. In science, the modularity principle is represented by search for basic elements (e.g., elementary particles, prototypes for different geometric structures...). In art, different modules (e.g., bricks in architecture or in ornamental brickwork...) occur as the basis of modular structures. In various fields of (discrete) mathematics, the important problem is the recognition of some set of basic elements, construction rules and an (exhaustive) derivation of different generated structures.

In a general sense, the modularity principle is a manifestation of the universal principle of economy in nature: the possibility for diversity and variability of structures, resulting from some (finite and very restricted) set of basic elements by their recombination. In all such cases, the most important step is the first choice (recognition or discovery) of basic elements. This could be shown by examples from ornamental art, where some elements originating from Paleolithic or Neolithic art are present till now, as a kind of “ornamental archetypes”. In many cases, the derivation of discrete modular structures is based on symmetry. Using the theory of symmetry and its generalizations (simple and multiple antisymmetry, colored symmetry...) for certain structures it is possible to define exhaustive derivation algorithms, and even to obtain some combinatorial formula for their enumeration.

3.2 Antisymmetry

Antisymmetry (or „black-white“ symmetry) is the symmetry of positive and negative, light and shadow... Antisymmetry introduced in ornamental art the possibility of expressing, in a symbolical sense, a dynamic conflict, duality, and illustrating alternating natural phenomena (day-night, tides, phases of the Moon, a change of seasons). Its domain can be extended to different geometrical properties, e.g., the relations of „convex-concave“, or „over-under“. Therefore, antisymmetry can be used for so-called *dimensional transition.* Treating the color change "black-white" as a space property, a suggestion of "two-sidedness'' (over-under, above-below) antisymmetry introduces a 3*D* space component in ornamental art. If you have an antisymmetrical („black-white“) structure in the plane, e.g., antisymmetric rosettes, friezes, or ornaments, their black parts can be considered to be placed under the plane, and the white parts over the plane. In this way, from the 2-dimensional symmetry patterns of rosettes, friezes, or ornaments, we can obtain 3-dimensional symmetry structures with their invariant planes and their corresponding symmetry groups of tablets, bands, and layers. Using contrast, complementary colors, "black-white", "light-dark", "over-under", "above-below", "positive-negative", "convex-concave", the same object can be turned into its opposite, increasing the rhythm and dynamics.

The idea of dimensional transition was the origin of the mathematical theory of antisymmetry. Visualization of symmetry groups of bands in a 2*D* plane, using black-white diagrams, was proposed by A. Speiser , and presented by L.Weber in 1929. The black-white diagrams of bands from his paper (Figure. 2), where the alternation of colors is used to denote figures above and below the invariant plane of the pattern, suggested the possibility for a more general dimensional transition from the symmetry groups of *n*-dimensional space, using the antisymmetry groups, to the symmetry groups of (*n*+1)-dimensional space. The natural idea of a more sophisticated dimensional transition from 3*D* to 4*D* space resulted in one of the first and the most remarkable early results of Heesch – the approximate number of four-dimensional groups preserving invariant 3*D*-space (less then 2000). The 1651 3*D*-

space antisymmetry groups, modeling the mentioned four-dimensional groups, were derived for the first time more then 30 years later by Zamorzaev in 1953. Unfortunately, the work of H. Heesh , as well as the paper of Woods giving the derivation of the 46 black-white symmetry groups of plane patterns, never attracted the attention of readers they deserved.

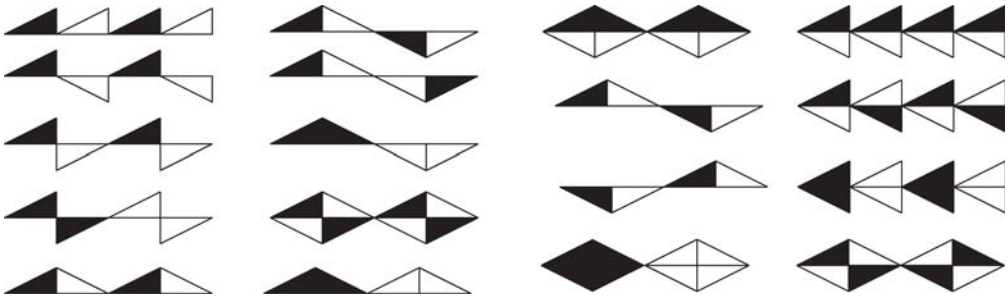

Figure 2: Black-white diagrams

The next generalization, theory of multiple antisymmetry introduced by Zamorzaev , is obtained if instead of only one bivalent property („black-white“, „over-under“...) we consider several bivalent properties commuting between themselve and with symmetries belonging to some symmetry group. As the result, we obtain multiple antisymmetry groups. From the point of view of tilings this means that a basic tile we can divide in *n* regions, and by its multiple antisymmetry coloring obtain 2^n states of the same tile (Figure 3).

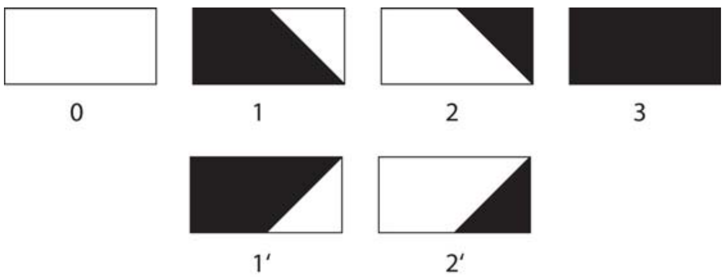

Figure 3: Antisymmetry coloring obtain 2^n states of the same tile

Multiplying such tiles by the multiple antisymmetry transformations, we obtain a very large number of different patterns. E.g. from rectangular tilings corresponding to the symmetry group pmm, by using numerical schemes derived by algorithm based on 2-multiple antisymmetry we obtain 2520 different patterns, and every change of the fundamental region results in a new series of 2520 patterns (Figure. 4). The obtained patterns have the following properties: 1) equal use of all basic tiles; 2) algorithmic creation of modular designs; 3) common visual identity of all designs. The same concept can be directly extended to 3D-structures (Figure 5).

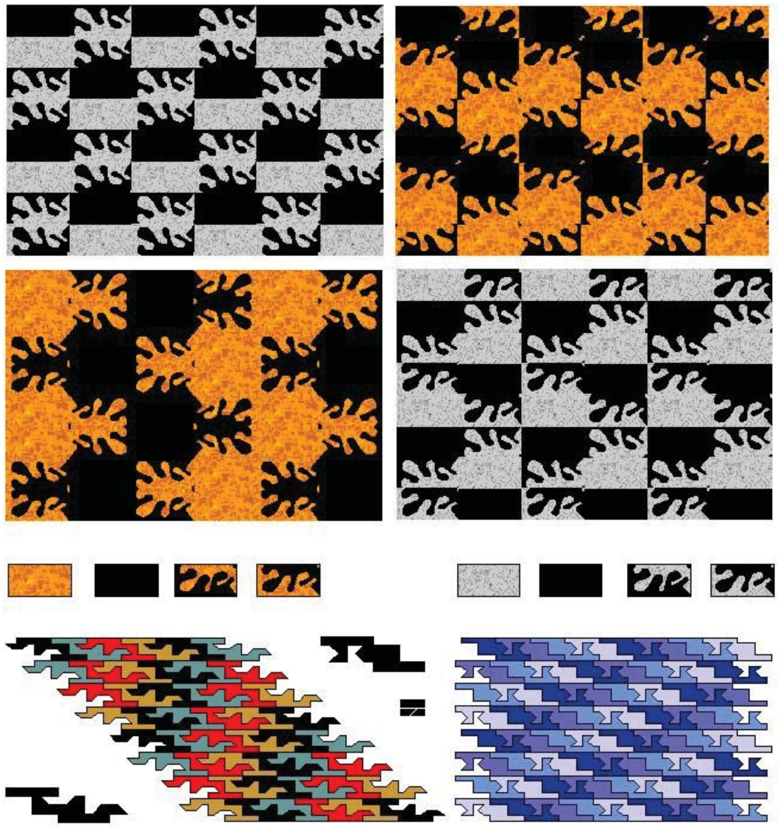

Figure 4: Algorithm based on 2-multiple antisymmetry has 2520 different patterns, and every change of the fundamental region results in a new series of 2520 patterns

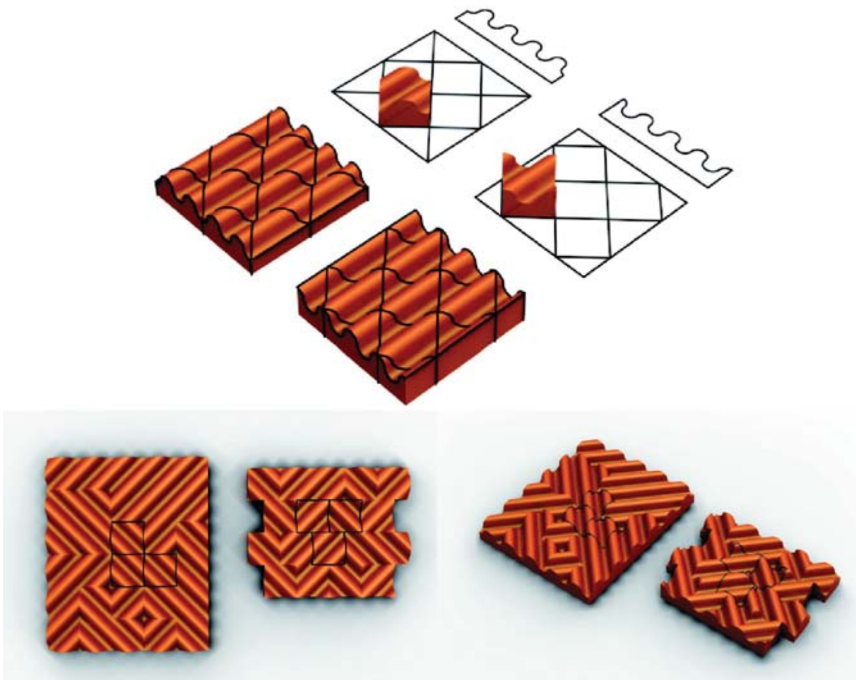

Figure 5: Extended 3-D pattern

4. From 2d to 3d ornaments

In our project, the treatment of wallpaper ornaments will be analyzed from the perspective of geometry. This means 2d ornamental motifs and color will not be the subject of the analysis; instead, they will only be used as the starting point for developing spatial ornaments. Spatial ornaments will be delineated using plane and space curves between which NURBS shapes will be generated. That way, the generated ornaments will represent basic shapes, modules that may be realized using the robotic arm.

The possible shapes of specific cells may be further analyzed based on the geometric structure presented in Figure 1. Symmetry groups *p1*, *p4* and *p3* will be used to analyze some possibilities of generating 3d ornaments.

The unit cell of wallpaper group *p1* may be a rectangle (Figure 6a) or a parallelogram, which is translated to make a group (Figure 6 b). In geometric terms, the cell has two pairs of different sides, which both delineate and separate the cells. That means that one pair (Figure 6c) or both pairs of sides (Figure 6d) may be replaced with arbitrary plane or space shapes.

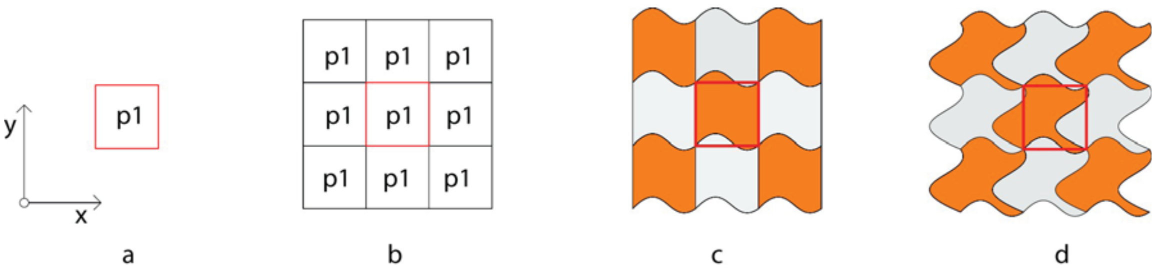

Figure 6: a. Unit cell of group p1; b. Basic structure; c. Change of shape of one pair of sides; d. Change of shape of both pairs of sides

Figure 7 shows one way to generate a 3d ornament. A NURBS surface is generated using plane curves (Figure 7d) and an arbitrary point in space ($x, y, z \neq 0$). As all curved sides lie in the *xy* plane, it is possible to mirror individual elements across the *xy* plane and thus additionally vary the symmetry group.

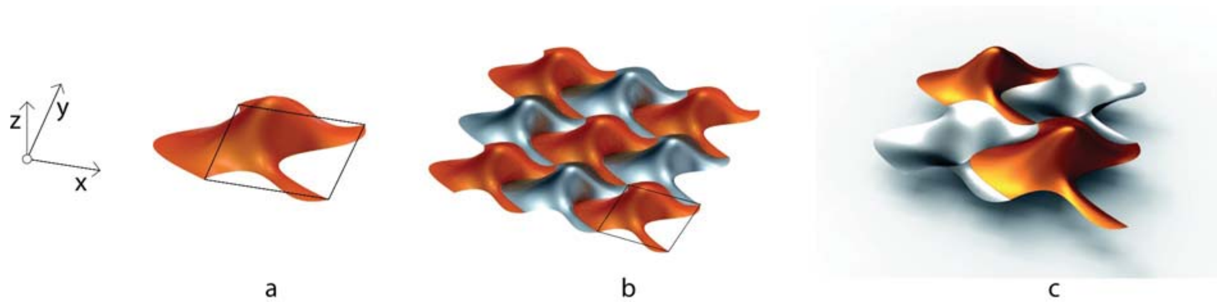

Figure 7: a. Spatial cell of group p1; b. Ornament configuration; c. Ornament variant (some cells mirrored across XY plane)

Figures 8 and 9 show the geometric structure of group *p4*. This group has two fourth-order rotation points and one second-order rotation point. In terms of geometry, the unit cell is lined by pairs of sides. The geometry of one pair is basic (Figure 8c), compounded by the same shape rotated by 90° (with the rotation point at the intersection of two curves).

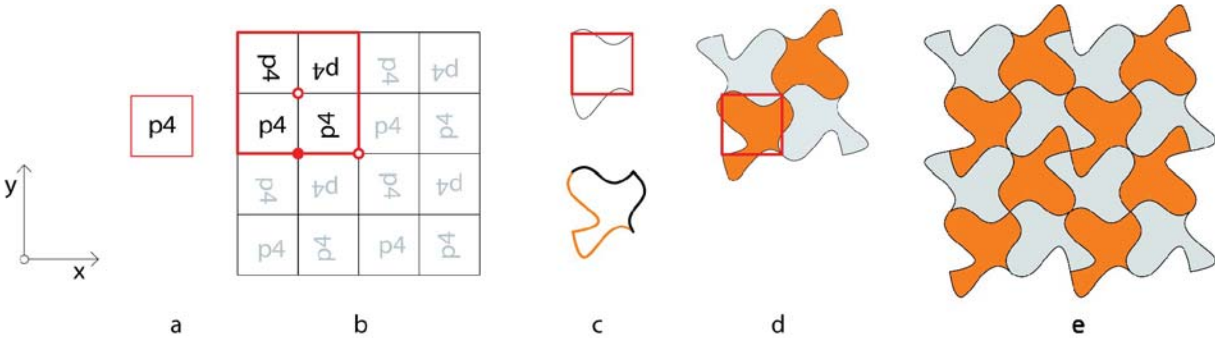

Figure 8: a. Unit cell of group p4; b. Basic structure; c. Changes to shape, with two curved sides rotated by 90 degrees and delineating the other two curved sides; d. Basic module of p4 ornament; e. Newly generated ornament

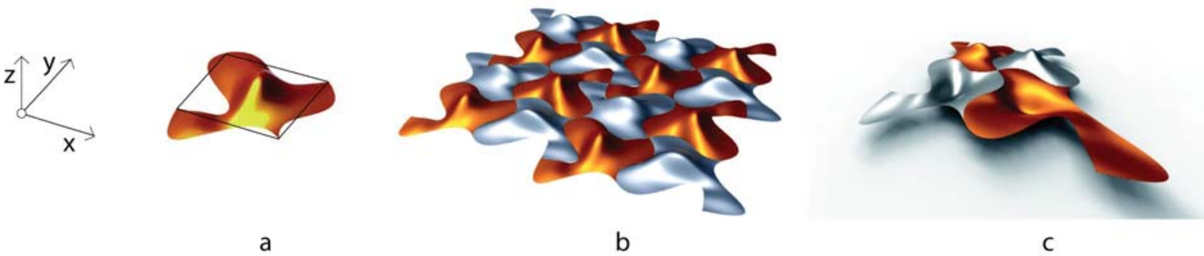

Figure 9: a. Spatial cell of group p4; b. Ornament configuration; c. Ornament variation (with some cells mirrored across XY plane)

Figures 10 and 11 show an example of transformation of a 2d ornament from group *p3* into a 3d ornament. Unlike previous cases, where the geometry of sides changed, the spatial geometry in this case is defined by single cell. Figure 10a shows a cell with spatial geometry/sides given in three different colors. This shape consists of four uniform prototypes whose position in space is special and which allow geometric continuity of adjacent elements.

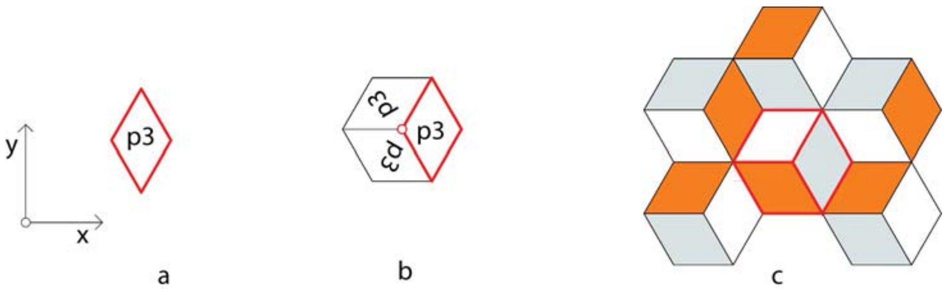

Figure 10: a. Unit cell of group p3; b. Basic structure; c. Ornament p3

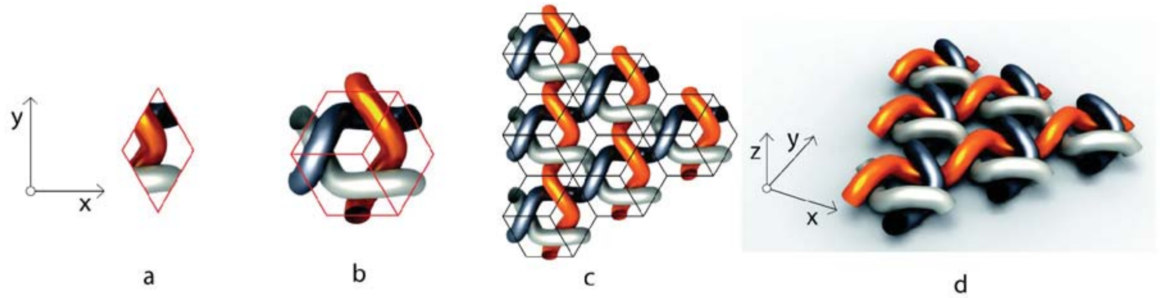

Figure 11: a. Unit cell consisting of four identical elements, of which two are orange, one is white and one gray b. Basic structure of ornament is made up of 3 spatial elements in 3 different directions c. continuity of structure in 2d d. Spatial configuration of ornament p3

The design of 3d ornaments based on mathematical laws of ornamental groups allows for plenty of creativity, despite being highly mathematically structured. The ways in which these ornaments may be transported into a parametric model and modular architectural elements were the topic of the course "Design of specialized topics" at the Faculty of Architecture in Graz.

5. Computer Serendipity – The Robot as the Architect's Best Friend

The course "Design of specialized topics" held in the summer semester of 2011 featured a project under the name "Computer Serendipity – The Robot as the Architect's Best Friend". The aim of the project was to investigate the possibility of transformation of 2d ornaments into 3d ornaments, generation of 3d digital models and their fabrication with the robotic arm ABB IRB140. The first objective was the definition of a simple cutting system for XPS/EPS elements that serve as molds for concrete elements, and the second objective was the definition of double curved elements that can be used as outer permanent wall insulation – XPS/EPS permanent concrete shuttering.
A special focus of the project was the efficiency of standard materials in the fabrication of non-standard elements. Namely, it is a well known fact that the fabrication of non-standard elements is connected with inefficient use of standard materials that greatly increases the costs of fabrication . This is why the process of design in this project started with geometry, modularity and ornamental rules. The students were introduced to the technical possibilities and limitations of the robotic arm, constraints when it comes to the cutting tool definition, limitations of materials, and geometric conditions that satisfy the requirement of material cost-effectiveness.

5.1 Design Approach

Since one of the project objectives concerned cost-effective use of materials, in the design phase the students started with modular ornamental parametrical elements. By varying their parameters and position, it was possible for them to get a huge number of variations of the final design. The big challenge for the students in this project was the optimization of the robot data flow from parametric design to a physical model. A basic Grasshopper file was generated within the project that included inverse kinematics with the necessary calculation for direct robot programming with an automatic code generated for the Robot ABB IRB140 (Figure 12).

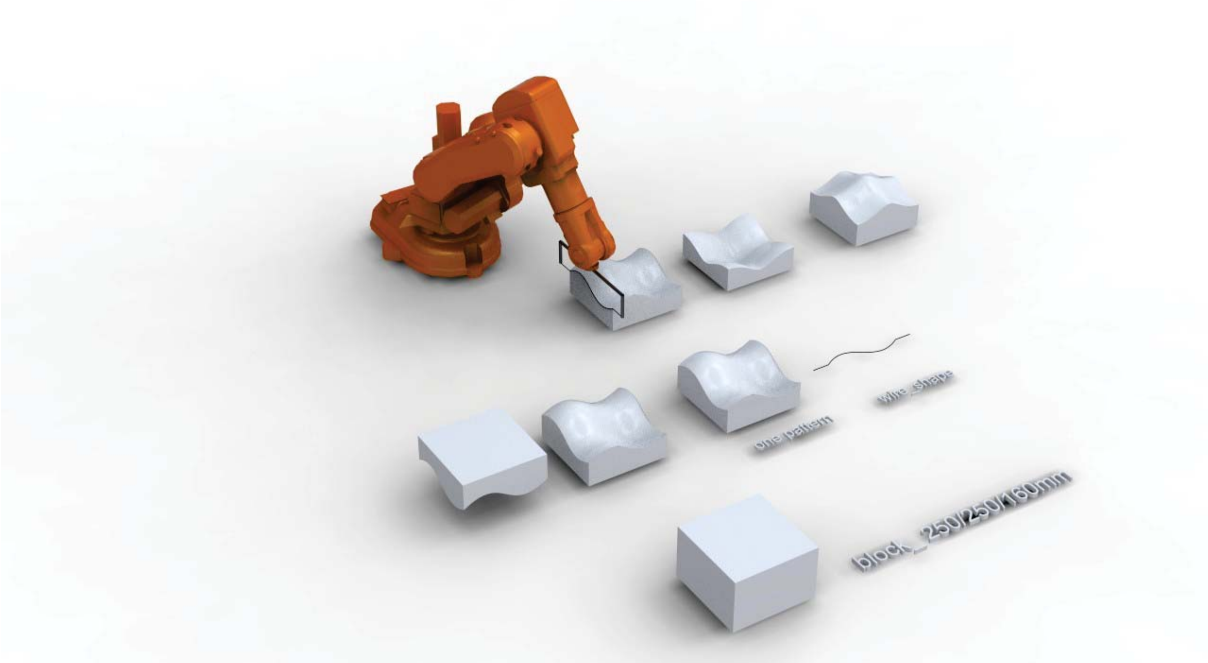

Figure 12: ABB IRB160 robotic arm and one of the tools used in our project for cutting EPS in modular parts

The basic robot code was supplemented with special strategies depending on individual specific design meant to facilitate the transformation of the desired design to the basic robot code. This approach provided great flexibility in design and real time feedback to the designer. The students used differently shaped hotwire and EPS panels (250x250x160mm) to cut into stackable ornamental components, which then composed a negative mold for a concrete wall design. Based on direct feedback, the students had the possibility to understand the robot's constraints, such as: size of the robot arm, availability of all desired positions regarding the chosen tool and robot axes restriction. These constraints necessitated the adaptation of individual designs, starting with changes to the initial design, selected tool or EPS tool cutting strategy.

Conclusion

The digital possibilities of generating 3d models and fabricating architectural building elements have led, among other things, to the re-establishment of ornaments in architectural practice. This paper shows the transformation of a 2d ornament into a parametric 3d ornament. This approach is based on a combination of mathematical algorithms and various linear and spatial shapes, where all geometric properties of mathematically defined ornaments remain invariant in the process of design.

The paper starts with the mathematical model of wallpaper symmetry groups and analyses the geometric invariants of specific ornamental groups and some possibilities of transformation of 2d ornaments into 3d ornaments. The process which starts with a geometric model and leads to design and one way ornaments can be used in architecture is presented using a selected student project, where students used mathematical algorithms to make their designs and export the desired design directly into the robot code, which was used for realizing individual elements.

Students' works

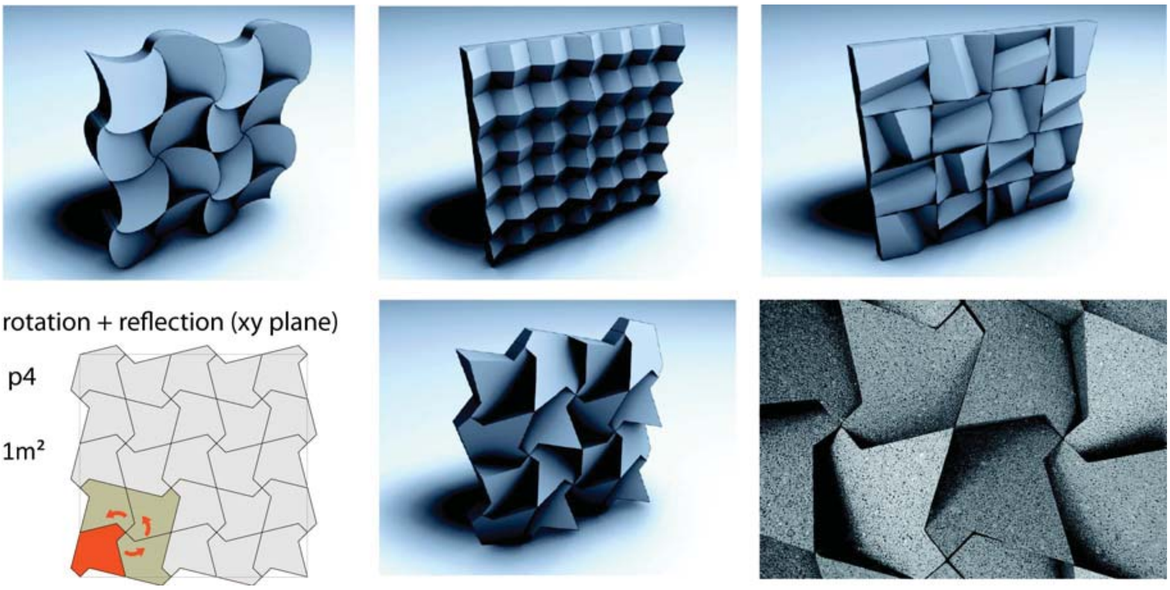

Figure A: Students: Florian Maroschek and Irina Elisabeth Scheucher

- creating an ornament, which can be poured of concrete - form has to be a negative.
- the ornament should slightly fit into the EPS block, which is going to be the form board.
- one piece fills 0,04m², consequential 25 pieces have to be build and joined for $1m^2$.

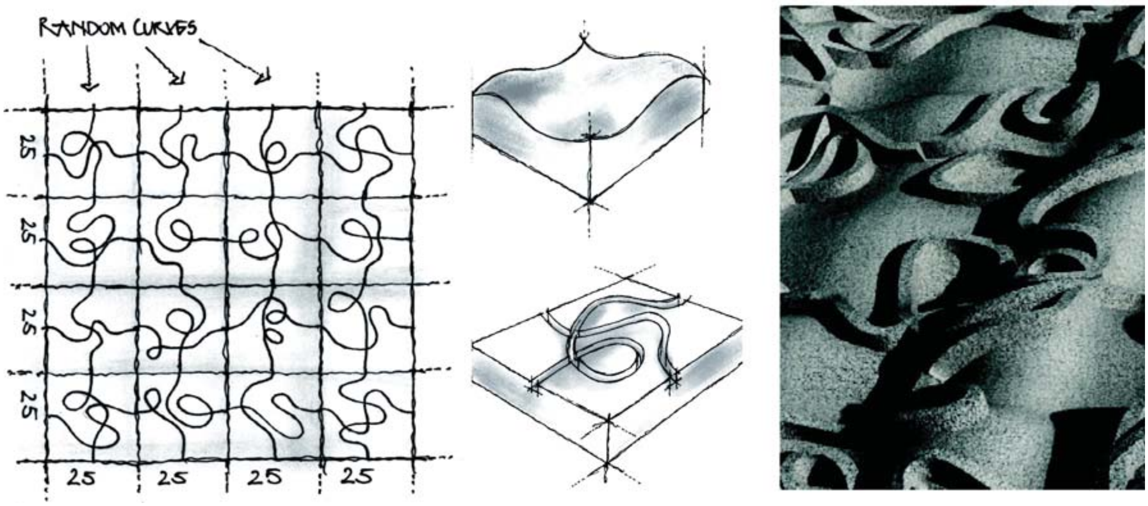

Figure B: Students: Wolfgang Moritzer I David Längle I Klaus Weber

- *surface design: surface was generated by two same curves – on that way each block can be arranged together with any others.*
- *curves: all curves are on the surfaces and each curve is a random curve with first and last points at the middle point on the block boarder*

Figure C: Students: Christopher Leitner and Thomas Hörmann

- *pattern deals with the material specific characteristics - the compactness and hardness of an concrete wall and on the other hand the smoothness and flowing which is always associated with concrete*
- *the design allows a multiplicity of capabilities in an architectural context.*
- *from small scale use like a bar or a dividing wall (interior design) to large scale use like facade elements. The form is capable to absorb vertical forces (statics) as well as it can be used as a sunshield.*
- *translational surfaces with 4 different pattern (but only one cutting wire)*

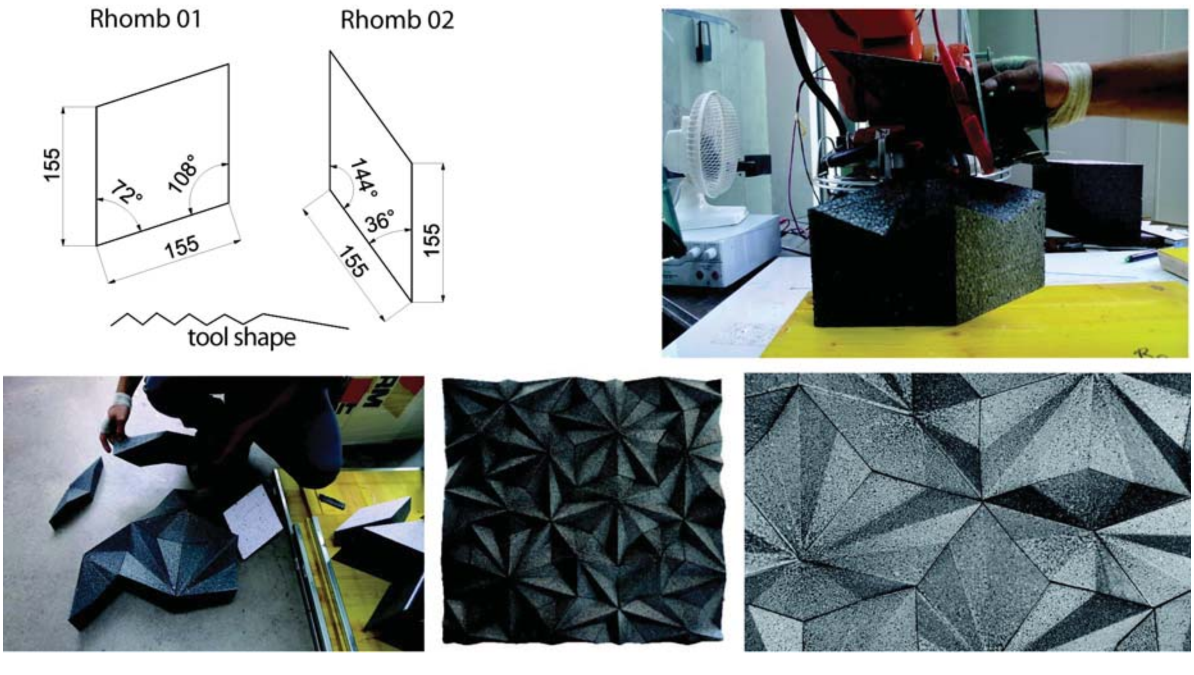

Figure D: Students: Georg Schrutka and Stefan Rasch

- *aperiodical pattern based on the Penrose tilling*
- *two different bricks in the form of rhombuses, one with corner angels of 72° and 108°, second with corner angles of 36° and 144°*
- *The user is able to combine these bricks to get an individual large area pattern.*
- *The shade of the edges is arbitrary. The heights of the corners of the two rhombuses have to be the same.*

Acknowledgements

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References

[1] Fröhlich, M.: 1991, *Gottfried Semper*, Studiopaperback, Artemis & Winkler Verlag, Zürich and München.

[2] Hauer, E.: 2004 *Continua - Architectural Walls and Screens*, Princeton Architectural Press.

[3] Heesch, H.: 1930, Über der vierdimensionalen Gruppen des Dreidimensionalen Raums, Z. Krist., 73, p.p.325-345.

[4] Hekkert, P.: 2006, Design Aesthetic: Principles of Pleasure in Design, *Psychology science*, http://studiolab.io.tudelft.nl/hekkert/publications.

[5] Jablan S. : 2002, *Symmetry, Ornament and Modularity*, World Scientific Publishing, Singapore

[6] Jansen and Conway, P.:1982, *The new decorativeness in architecture & design*. Foreword by Paul Goldberger, Clarkson N. Potter, New York.

[7] Leder, H.: 1995, Wem Schönheit nützt. Psychologische Ansätze zur Ästetik. Antritts-vorlesung von Univ- Prof. Dr. Helmut Leder am Freitag, 18.März 2005, Universität Wien, Online-Zeitung der Universität Wien, www.dieuniversitaet-online.at.

[8] Loos, A.: 1997, *Trotzdem, Gesammelte Schriften 1900-1930*, reprint, Georg Prachner Verlag, Wien.

[9] Moussavi, F. and Kubo, M. (eds.): 2006, *The function of ornament*, Harvard University Graduate School of design.

[10] Shubnikov, A.V. and Koptsik, V.A.:1974, *Symmetry in Science and Art*, Plenum, New York.

[11] Speiser, A.: 1927, *Theorie der Gruppe von endlicher Ordnung*, 2. Auflage, Springer Verlag, Berlin.

[12] Venturi, R.: 1966, *Complexity and Contradiction in Architecture,* Published by Museum of Moder Art, New York.

[13] Woods, H.J.:1935, *The geometrical basis of pattern design*, J. Text. Inst., 26, p.p. 197-293.

[14] Zamorzaev, A.M.: 1976, *Teoriya prostoi i kratnoi antisimmetrii*, Shtiintsa, Kishinev.