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Paper : **FROM LABYRINTHS AND RECURSIVE FOLDS TOWARDS GENERATIVE ARCHITECTURE**

Abstract:

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This labyrinth, as all those found in medieval churches (which are borrowed from those depicted in antiquity) is a unicursal (non-branching and without dead ends) path, that leads without ambiguity from the entry towards the centre.
Such a classical labyrinth may be considered as a folded line that tends to be of a maximal length in a definite area, i.e. a line (1D) that tends to fill a part of a surface (2D), and thus may be linked to FASS (space-**F**illing, self-**A**voiding, **S**imple and self-**S**imilar) curves, and other recursively folded curves.
This paper discusses issues involved in labyrinths as well as FASS curves, and in the relationship between them.
Then it explains ways to make labyrinthine FASS curves or other recursively folded curves, especially through edge-rewriting and node-rewriting L-systems, in different spaces (2D, 3D or fractal) and various shapes.
Finally, implications and uses of the labyrinth, and its extrapolations, in generative architectural design are suggested and explored.

Topic: Architecture

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Finger labyrinth in Lucca Cathedral

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From Labyrinths and recursive Folds towards generative Architecture

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Abstract

On a pillar of the portico of Lucca Cathedral is carved a «digital» labyrinth, i. e., a labyrinth that is to be followed by the mean of a finger.
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Finally, implications and uses of the labyrinth, and its extrapolations, in generative architectural design are suggested and explored.

1. Labyrinths

1.1 What is a Labyrinth?

In any language, the current meaning of the word *labyrinth* suggests some complex structure, through which it is difficult to find one's way, and from which it is difficult to escape. The streets of a medieval town or the corridors of a complicated castle are said to form a *labyrinthine* network. Metaphorically, a nightmarish situation in which one has to find one's way through abstruse rules, knocking on non-opening doors, puzzling on unanswered questions, like Joseph K. in *The Trial* [1], is qualified as *labyrinthine* as well. According to this colloquial use of the term, a labyrinth would be a *multicursal* (i. e. branching, with dead ends and even loops) pattern of paths.

Fig. 1: Finger labyrinth in Lucca

However, such is not the labyrinth engraved on a pillar of the portico of Lucca Cathedral (Fig. 1). Its path, meant to be followed by the finger (and which is not the engraving itself, but is defined by it), is *unicursal*, leading without ambiguity from an entry at the right of the perimeter towards the centre. The way back out of the labyrinth poses no problem, and there would be no need of a thread to find it. Its pattern is identical to most ones of medieval labyrinths, of which the one in Chartres Cathedral is an archetype (Fig. 2).

Fig. 2: Chartres labyrinth

Most are, like the Chartres one, pavements on the floor, and probably meant to be followed by pilgrims on their knees.

Practically all of those medieval labyrinths have the same pattern, which is the one drawn by Villard de Honnecourt himself (Fig. 3). Most are circular patterns, though octagonal or even squared variants may be found.

Fig. 3: Sketchbook of Villard de Honnecourt (about 1230)

The reference to the myth of the Minotaur in medieval patterns, even when it is not explicit as in Chartres, where a copper plate depicting Theseus, Daedalus and the Minotaur was in the centre of the labyrinth), is obvious and well documented. The medieval labyrinth pattern itself is however different from ancient labyrinth patterns like those shown in Figs. 4, 5. This pattern is mostly round, though there are some squared ones, like the earliest recovered one, incised on a clay tablet from Pylos (Fig. 4).

Fig. 4: Labyrinth incised on clay (Pylos, Greece)

Fig. 6: Cretan coin (British Museum)

That is the same ancient pattern that is used in Scandinavian so-called «Troy Towns», which are medieval labyrinths made of stones (Fig. 7), dated from as far back as the 13th century.

Fig. 7: «Trojeborg», or «Troy Town», a stone labyrinth (Sweden)

In both (ancient and medieval) cases, a labyrinth is a pattern that defines a unicursal path, but it must be noticed that in any case what is drawn, engraved, built, and so on, is the pattern of «walls», i.e. the pattern (which is itself multicursal) from which the path, sometimes called «Ariadne's thread» (though one does not need a thread to find one's way out) is deduced. This path runs around the centre (with twists) a certain number of times, called rings. The number of rings differentiates variants of the pattern from one another. The classical ancient pattern comprises 7 rings (Fig. 8), while the medieval one is made of 11 rings (Fig. 9).

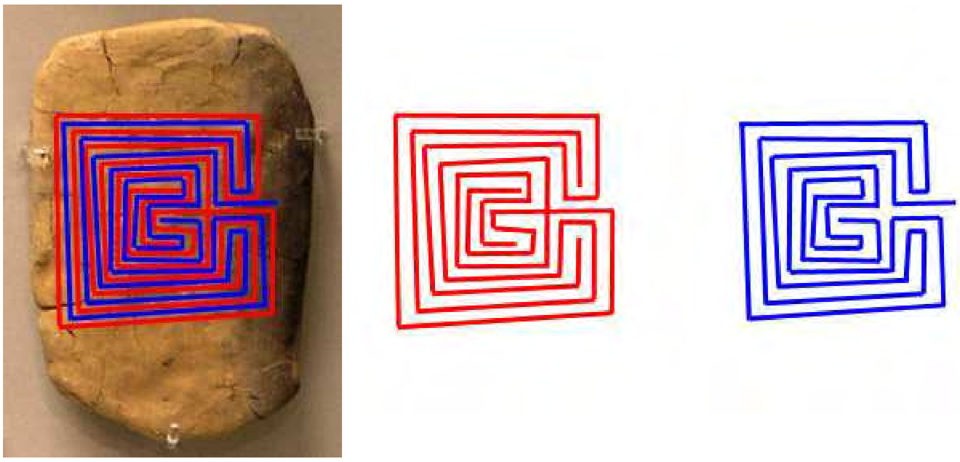

Fig. 8: Walls and path of the ancient labyrinth

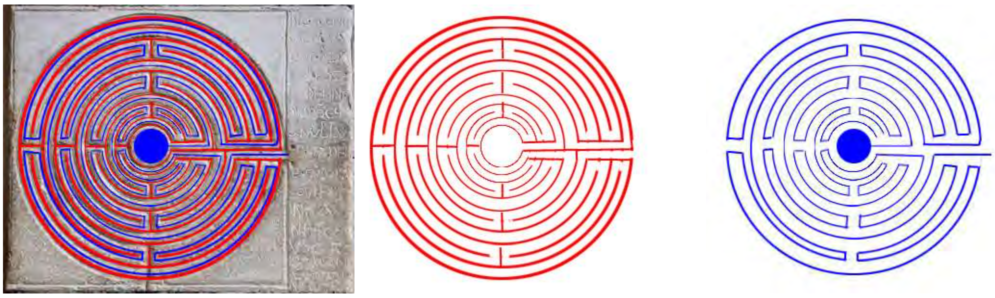

Fig. 9: Walls and path of the medieval labyrinth

One may notice that all these patterns (of walls) show a straight wall that goes from the perimeter towards the centre. In all those patterns the path begins on one side of this radial wall and ends on the other one. From this wall emerge two «branches», right across each other for the ancient pattern, shifted for the medieval one. This observation leads us to the construction of the labyrinth.

1.2 How to construct a Labyrinth

Hermann Kern, in his «bible» of the labyrinth [2], explains how to construct a 7-ring ancient pattern of labyrinth. You start with a cross, and you draw L shapes inside the corners of that cross, and put a dot inside each L. Then, the process is very straightforward: you join the top of the cross with the top of the L at its left, and you join what you find following on each side, making some round ring around the pattern in progress. That is how you obtain a 7-ring ancient labyrinth pattern (Fig. 10)

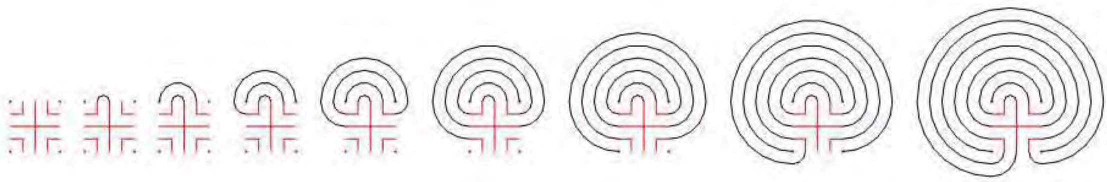

Fig. 10: Construction of the 7-ring ancient labyrinth

Variants with 3 or 11 rings are produced by eliminating the Ls or doubling them respectively (Figs. 11,12). You can get two variants of 5 (resp. 9) rings by eliminating (resp. doubling) the top Ls or the bottom ones (Fig. 13)

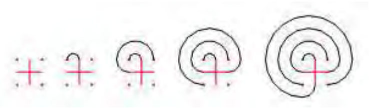

Fig. 11: Construction of the 3-ring ancient labyrinth

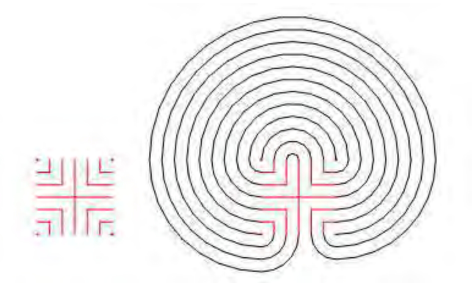

Fig. 12: First and last steps of the construction of the 11-ring ancient labyrinth

Fig. 13: Construction of the two variants of a 5-ring ancient labyrinth

All those patterns consist of an odd number of rings. Such are indeed most, or maybe all, of the labyrinths actually found. But the method described above may lead to even numbers rings. One has simply to eliminate one L in the starting scheme of any of the schemes already used. This leads to patterns in which the path begins and ends at the same side of the radial wall.

In that way, we could obtain a series of patterns, with 1, 2, ..., n rings.

We have noticed before that the perimeter of the ancient pattern is not a circle, but rather some sort of spiral. One step towards the medieval pattern is to «circling» the ancient pattern, and to slightly change the method, by starting with a «cross» with shifted arms, and drawing parts of concentric circles when joining dots. That is not a medieval pattern yet, but it is this pattern that we shall use in our further experimentations.

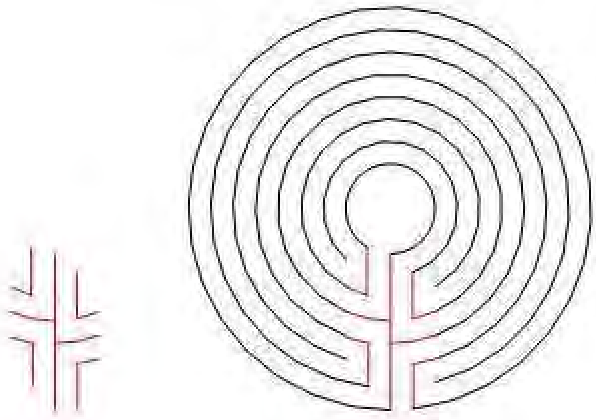

Fig. 14: Construction of the 7-ring ancient pattern, with rings on concentric circles

The medieval pattern is more elaborated: it has got partial radial walls in three other directions, blocking two rings at a time, and forcing the path to turn back. But by replacing the Ls by some sorts of «double» Ls, and inserting radial barriers at the right places, we can deduce the construction of the 11-rings medieval pattern from that of the 7-rings ancient pattern.

1.3 Generating the labyrinth

The presence of the radial wall lets us «cut» the pattern and imagine that we «spread» it, in order to obtain a rectangular pattern (Fig. 15). That transformation may be geometrically expressed as a change of coordinates, from polar coordinates, to euclidean ones.

Fig. 15: Spreading the 7-ring ancient pattern

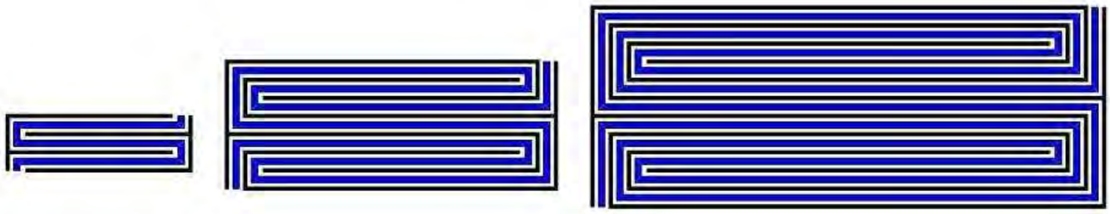

Fig. 16: Spread 3, 7, and 11-ring ancient patterns

This representation of the labyrinth pattern, and especially of its path, lets us better understand what is actually a labyrinth. It has been said that a labyrinth is not a meander (also called «Greek key»), but the unfolded path of the classical ancient 7-ring labyrinth clearly shows a double meander. The meander is not a spiral, but it is in a way a «double» spiral, a spiral that enters doubled by a spiral that goes out.

Spread out, and so in a way unfolded, the labyrinth path is still a folded line, and even a recursively folded line. The progression in the number of rings lets us imagine a process in which we would write a rule of transformation to get from pattern to pattern.

Considering the meander, one can translate its progression by this L-system:

L-system #1
V={X,A,B,F,+,-}
ω : B-X+B
X → AFFF-A-X+A+AFFF
A → AFF
B → AF
F → F

The interpretation of the symbols is as such:
A, B, F : move forward (and draw a line)
X : move forward (and draw a line) three steps
+ : turn left
- : turn right

The length of each step is the same, the angle is 90°. Steps 1 to 5 of the derivation are shown in Fig. 17.

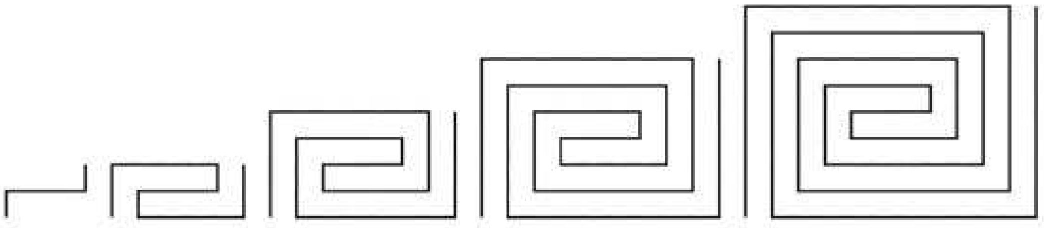

Fig. 17: Steps 1 to 5 of the derivation of L-system #1

The ancient pattern is made of two meanders, its 3, 7 and 11-ring variants may be easily deduced from the three first step of this L-system.

One can also spread out the medieval path pattern. Unfortunately it is difficult to translate this pattern into some sort of L-system. Anyway this pattern reminds us of some of those encountered in FASS curves.

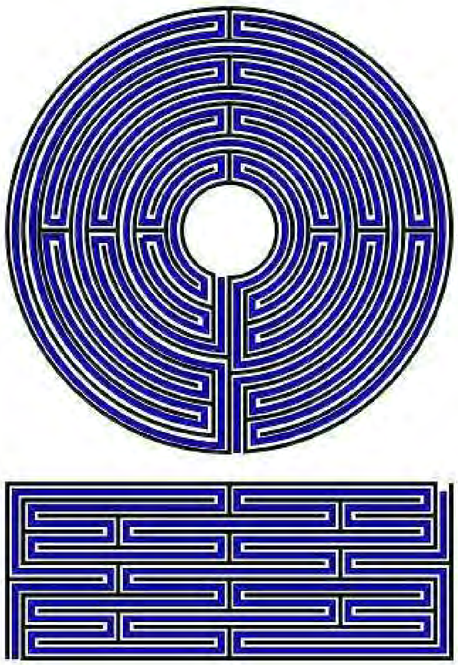

Fig. 18: Spreading the medieval pattern

2. FASS curves

2.1 What are FASS Curves?

FASS (space-**F**illing, self-**A**voiding, **S**imple and self-**S**imilar) curves have been known as early as the end of the 19th century, and belong to that «gallery of monsters» Mandelbrot refers to when he is forging his fractal theory [3]. The most known ones are the Hilbert curve (1891) and the Peano curve (1890).

The aim of those mathematicians was to exhibit paradoxical objects, in order to prove that what some of their colleagues thought impossible, was actually existing. They exhibited curves (dim. 1) that could be assimilated to surfaces (dim. 2), as they pass through every point of them and contributed as such in the set theory regarding issues of dimension and infinite.

Some of those mathematical paradoxical sets were obtained by recursive holes through a segment (Cantor set, 1883), a triangular surface (Sierpinski gasket, 1915), a cube (Menger sponge, 1926). What characterises Hilbert and Peano curves, and all FASS curves imagined since those forerunners, and links them to the labyrinth, is that they are r*ecursive folds*.

2.2 How to get FASS Curves by L-Systems

FASS curves inscribed in a square may be considered as edge-rewriting or node-rewriting L-systems [4]. In any case, one must take care of the definition of such a curve, as a finite, *self-avoiding* approximation of a curve that passes through *all* points of the square.

In edge-rewriting L-systems, one has to consider the square recursively divided into 2 x 2, 3 x 3, 4 x 4, or more generally n x n tiles, and to find a path through all the points of the initial grid. This path must be self-avoiding, but the replacement of each tile by this path must also yield a self-avoiding line.

As it is an edge-rewriting system, the start and end points of the initial path must be at adjacent vertices of the square. Once the path is found, one has to consider its inverse, and to replace each edge by the initial path or its inverse, depending on the side on which the replacement must take place. By experimenting with 2 x 2, 3 x 3 or 4 x 4 tiles the exploration of all possible paths shows that the condition of being self-avoiding (the path must not touch itself neither by an edge nor by a node) is not reachable.

Actually, it has been demonstrated that the simplest FASS curve obtained by edge replacement in a square grid is the so-called E-curve, which requires a 5 x 5 grid.

In node re-writing systems, one has also to consider a tiling of the square, but as it is the node, and not the edge, which will be replaced, one can consider either a path that links two adjacent vertices of the initial square, or two diagonally opposed ones. The chosen n x n grid leads actually to a n+1 x n+1 tiling, because that is the nodes that are replaced, and there are one more node than edges (on the side of the initial grid).

For the simplest grid we can imagine (1 x 1), there is only one way to find a path through the four vertices of the square, a U path, and it links two adjacent vertices of the square (there is no way to link the diagonally opposed ones, obviously). Once one has determined the left and right position of each replacement, the corresponding L-system is straightforward.

L-system #2
V={L,R,F,+,-}
ω : L
L → +RF-LFL-FR+
R → -LF+RFR+FL-
F → F

The interpretation of the symbols is as such:
F : move forward (and draw a line)
L and R are not interpreted but only derived
+ : turn left
- : turn right

This simplest FASS curve is actually the Hilbert curve mentioned above.

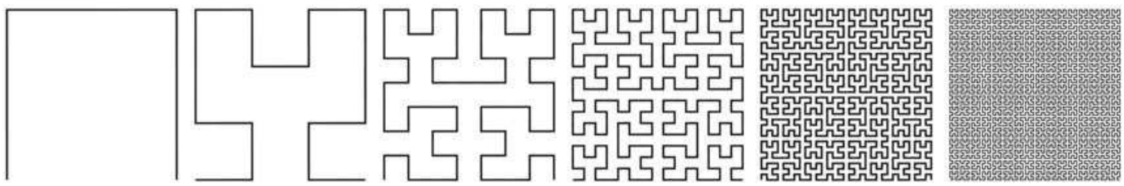

Fig. 19: Steps 1 to 6 of the derivation of L-system #2 (Hilbert curve)

Now, starting from a 2 x 2 grid, there are only two possible paths, the first one being a S path linking tow opposite vertices. This node-rewriting L-system leads to the Peano curve mentioned before.

L-system #3
ω : L
L → LFRFL+F+RFLFR-F-LFRFL
R → RFLFR-F-LFRFL+F+RFLFR

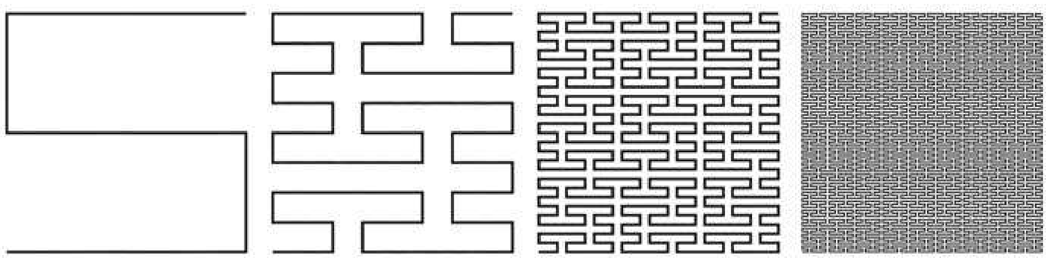

Fig. 20: Steps 1 to 4 of the derivation of L-system #3 (Peano curve)

The other path, linking two adjacent vertices of the square, leads to the other FASS curve shown in Fig. 21.

L-system #4
ω : L
L → LF+RFR+FL-F-LFLFL-FRFR+
R → -LFLF+RFRFR+F+RF-LFL-FR

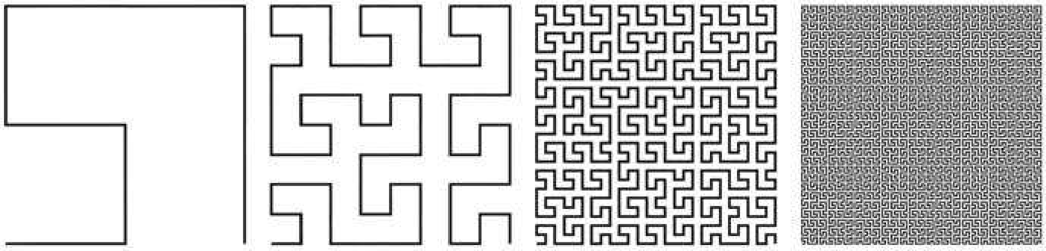

Fig. 21: Steps 1 to 4 of the derivation of L-system #4

Considering the subdivision of the square into more tiles, we can generate many other FASS curves. Among them, one is especially interesting for our topics, as it begins with a path through the 4 x 4 square which is actually a meander (Fig. 22)

L-system #5
ω : L
L → LFRFLFRFL-F-R+F+L-F-RFLFRFL-F-RFLFR+F+LFRFLFR+F+L-F-R+F+LFRFLFRFL
R → RFLFRFLFR+F+L-F-R+F+LFRFLFR+F+LFRFL-F-RFLFRFL-F-R+F+L-F-RFLFRFLFR

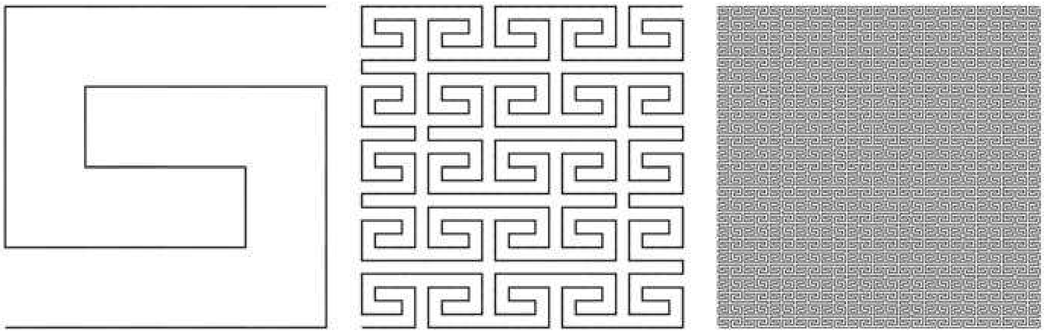

Fig. 22: Steps 1 to 3 of the derivation of L-system #5

2.3 Transforming FASS Curves into Labyrinths

A first comment we can make when looking at those FASS curves and comparing them to the labyrinth path patterns, is that the tiling entails some round-turnings, which would correspond to intermediate «walls» in terms of labyrinth patterns. Except for their first derivation, they are then essentially different from the ancient labyrinth pattern.

On another hand, classical labyrinth patterns, and especially the medieval archetypal pattern, involve that the path begins on one side of the separating wall and ends on the other, which corresponds for the FASS curve scheme to a path that links diagonally opposed vertices of the initial square.

Anyway, one can imagine new labyrinths by doing the inverse operation we did in spreading out the labyrinth in 1.3. This transformation (which acts on a bitmap representation of the FASS curve) changes the coordinates from euclidean to polar.

Using some representation of step 2 of the Peano curve (see Fig. 20), and transforming it, we obtain something that could be followed like the finger labyrinth in Lucca.

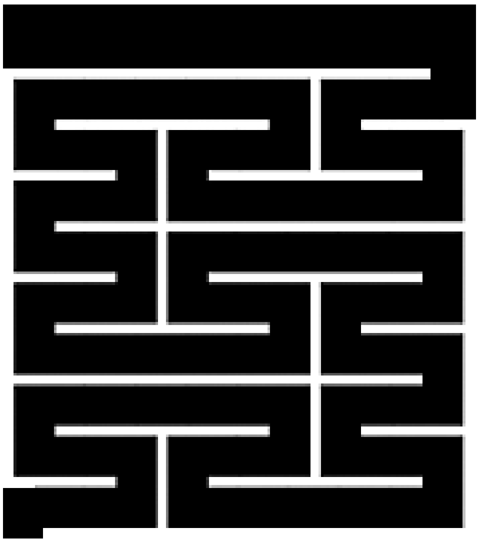

Fig. 23: A representation of step 2 of the Peano curve

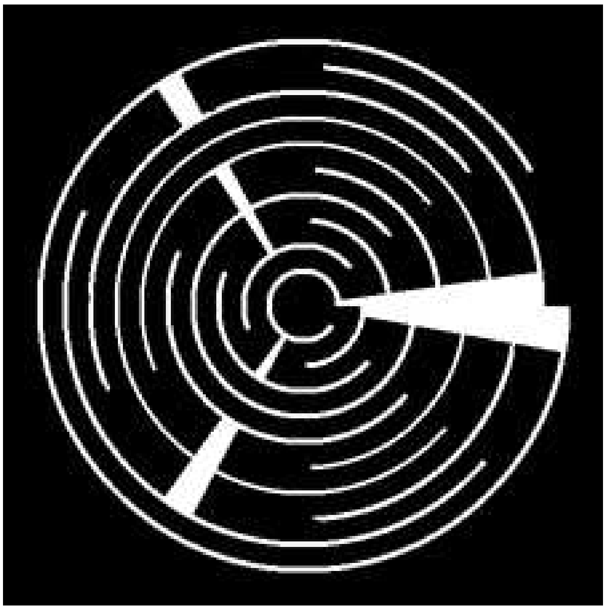

Fig. 24: Transformation of Fig. 23 through a change of coordinates

Fig. 25: An interpretation of Fig.24 as an engraved stone

3. Provisional conclusions and further questioning

A labyrinth is a path, a route, and some authors ascribe its origin to a ritual dance. The *path* is one of the interpretations of the line, the other one being the *limit* (the border of a surface).

We can see two fundamentals characteristics of the labyrinth as a path that links the perimeter of a shape to its centre: it is a mean of disorientation, and it is a way of putting a very long path inside a bounded area.

The labyrinth is not a spiral, it changes often of direction and then tends to disorient the traveller. It moreover offers a deceiving hope of reaching the aim, as it diminishes and increases the distance from the centre all the time. In the medieval pattern, one reaches the ring nearest from the centre very soon, and has to go on the farthest at the end of his travel.

The labyrinth as a way to put a very long path inside a relatively small surface is illustrated by the length of the Chartres labyrinth (261.55 m) inscribed in a circle of diameter 16.40 m.

The labyrinth is one of the fundamental myths of origin for architecture, and Daedalus, its inventor, is said to be the father of all architects.

Leaving apart the ambiguity of the definition of the labyrinth, one may wonder why a labyrinth, which is fundamentally a path, is so important in architecture. There are actually cases in which the path is the essential part of a design. One can think of the pattern involved in such Swedish furniture store, or, more architecturally interesting, of a museum.

Le Corbusier chose the spiral for his «Musée à croissance illimitée» (1939), but he could have explored some form of labyrinth.

Frank Lloyd Wright also chose the spiral for the Guggenheim Museum (1959) in New York, or rather an helix, as the spiral rises up.

The labyrinth is fundamentally a 2D pattern, distinct from the interlace, which, even if it is drawn on a plane, supposes a third dimension. However, one can try to imagine what the labyrinth concept means in 3D, by relating to FASS curves. In order to conceive such 3D FASS curves, one has to find a path passing through all the vertices of a 3D grid dividing the cube.

For instance, in the same way as the Hilbert curve fills a square, a very well known FASS curve fills a cube. This Hilbert 3D curve is shown Fig. 26.

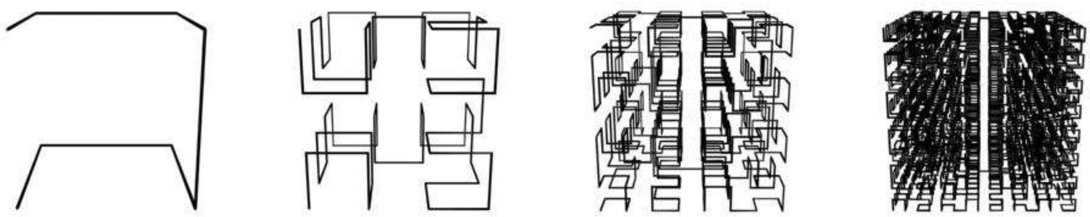

Fig. 26: 3D Hilbert curve

One line of further questioning would be about a possible 3D transposition of the labyrinth.

But, more essentially, the importance of the signification of the labyrinth in architecture should be detected and questioned.

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