Generative Art Geometry.
Logical interpretations for Generative Algorithms.

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Abstract

This paper tries to identify the creative processes of Generative Art that brings to the construction of dynamic procedures of transformation, generative algorithms, by departing from interpretative logics. This construction becomes possible through a dynamic approach to Geometry. In fact, overcoming the logic of the figures and related rules, this approach opens to the logic of the progressive processes and to the dynamics of transformation inside the geometric space.

This dynamic use of Geometry can be performed crossing again the revolution operated by Brunelleschi, by Piero della Francesca and by Leonardo da Vinci. This Renaissance revolution founds on the convergence between Art and Science and on the discovery of the Perspective Logic.

The "formella" of Brunelleschi interpreted by P.A.Rossi indicated that Brunelleschi made a peculiar, not casual choice of a point of view, with a distance from Battistero equal to the side of a cube involving the architecture and the optic cone, indicated by the circle, able to have a correct perspective. This was the approach for defining the structure of perspective the "perspective tool".
Paolo Alberto Rossi, "La scienza nascosta", (the hidden science).

Quoting Decio Gioseffi, "The perspective has been the first mathematical (in systematic and univocal terms) formalization of a "physic" law indefinitely "extensible", of general validity and general verifiability". The perspective, also in the first geometric tools structured by Brunelleschi, is a logical form of representation of the space that allowed, for the first time in human culture, to represent the infinite. The Perspective performs the representation of the infinite identifying a point of view. This means that the complexity of the space is
scientifically investigable through the subjectivity of an observer and his Logical Interpretations. The scientific search, in fact, can follow too the same interpretative way pointed out by the perspective. Until now, as shown by Einstein and his logical interpretation of the universe through the theory of Relativity, together with Max Planck and his quantum theory that is a different logical interpretation of the universe. Both theories are useful and true, also if so in contrast one each other. The points of view are different but the matter is the same.

Generative Art pursues this interpretative approach. And it does it redrawing its tools starting from the main one, the Geometry. The interpretative logics, activated by Generative Art, build parallel, multiple and progressive paths of dynamic transformations. These are managed through algorithmic logics. The Generative Geometry really becomes one of the main tool of Generative Art because it is able to logically represent the interpretation of the author performing his artworks in the endless multiplicity of the possible variations. The act of writing Generative Algorithms is representing and investigating the existing environment from different and progressive logical points of view, tracing the rules for transforming it from the past into the future.

The generative geometry
Geometry is one of the main fields involved in the construction of the generative algorithms. Not only for architecture, design and visual art, but also for music and poetry.
Since Generative Art moves from static forms to progressive transformations, Generative Geometry should be considered as the main tool for managing dynamic processes of transformation. Generative Geometry moves from geometric figures to the representation of dynamic logic processes, from measures to dynamic proportions, from measurable figures to measures related to a point of view, from representations of limited spaces to representations of infinity.

The transforming progressive process from Archaeopteryx to Apatornic following the Logical Interpretation of D'Arcy W. Thompson, "On Growth and Form", Cambridge Univ. Press, 1961. The Geometric structure is considered in analytical way following Durer, as a series of deformations. But I like to interpret, with generative geometry, his analytical tables, like the one in the image: a transforming process could be identified because the image looks like a
"perspective" representation.

Exemplifying, such potentialities could be represented by the passage from axonometric representations to perspective views, the only ones that logically represent the infinity. But not only. The Generative Geometry is much more. The construction of generative and geometric algorithms founds more specifically on logical interpretations of what fascinates us, by fixing our point of view. It's also a way to represent our main references, our preferred results of the past: the work of our main masters. Not copying them but interpreting them as results of a possible progressive process of transformation able to perform the quality that we appreciated. The aim is to construct procedures able to bring our design process in reaching such qualities.

Not analyzing these qualities but identifying which quality we like to transfer to our artworks, which quality corresponds to our own vision. This goal is performed by clearly identifying the point of view and the objective.

Operationally we are not doing copies of forms that interests us for the construction of a code, of a rule that represents our hypothesis: "how" we can construct events with the character that we like. And we will try to use these rules for managing the progression from the existing events to the possible ones; in other words for designing or making art. The logical-geometric interpretation of our imaginary of reference, of the works of our masters, of what fascinates us, is the core of the construction of a generative engine and of our creative tools.

In my generative design I have had a preference for the specific field of 3D space, also because my main sector of interest is architecture. But the Logical Interpretation of Geometry starts from one-dimension and two-dimension events as Kandinskij points out in "point, line and surface". We can find the more simple experimentations of using interpretative logics and managing the progressive dynamics when we construct lines through the generation of progressive points governed by rules. If our reference is the curve structured by Kandinskij, we can build an algorithm that defines, in progression, the following point through the progressive transformation of some parameters able to point out the verse, the dynamics of variation of the bending and the points of catastrophe where the direction suddenly changes, the progressive acceleration, the dynamics of variation of the thickness, etc.

We will never succeed in representing the famous line by Kandinskij (also because we don't like to copy it but to generate a kind of lines fitting similar aims) but we will produce a whole series of lines that represents the character that mostly interests us. The aim is to represent this characterized line with a transforming rule able to always turn a point into a different line but every time belonging to the same species of lines. So we have built a simple generative algorithm. And we have also represented an "ideal" line as a whole possible dynamic representations of a point in relationship to the precedents and the following ones. An Idea is "generatively" represented only when this "representation" can produce endless variations of the same event, all belonging to the same character. As, in Nature, a sequence of very different olive trees are all recognizable as olive tree. Variations are infinite because there is no limit to variations of individuals belonging to a species, of representations of the same objects belonging to the same logical interpretation but changing the point of view.

Increasing the complexity of our approach and moving over the simple one-
dimension geometry, we can build other algorithms able to define other dynamics of transformation. We can use them in the transforming process from a point to a line, from a surface to a solid, but also in each possible process from a dimension to the following one.

Remaining on two-dimensions, if we, for instance, have as reference the refraction of the light in a prism of glass, we can write an algorithm that, when our progressive line meets another line with particular colour, it defines how it breaks in a series of divergent lines that, after the "impact", will have autonomous life.

But, as it appears obvious, we are already moving toward an increase of dimensions. The acceleration already points out another dimension that can be represented in various ways.

The simpler three-dimensional generative process is the logic of cellular automata, when this kind of process is activated in the three-dimensions.

It is difficult to imagine the final result of these progressions even if we can foresee of it, but we can predetermine its character: nothing is left to random and all depend on the spatial topological location of the first events and of the adopted rules.

We can talk, in this case, of a progressive logic, of a first kind of generative approach to geometry. But it foresees an intrinsic difficulty to manage own spatial vision and the characters of each possible result. For doing that it is necessary to experiment and to find connections among the adopted rules and the character of the results. This search is possible because the logical sequence of the transformations is fully controlled by the rules. Even if we can surprise of unpredictability, and sometimes of the unexpected beauty of the results, this happens without using logical random but only varying the mutual initial positioning of the events.

And here a fundamental aspect of generative processes appears: the use of random parameters. Firstly we need to clarify that the use of random for the initial data as the positioning of the first events in a process of cellular automata or the first points in the construction of lines through the logical progression of points, are really different from the creation of random forms and the subsequent choice of the form that casually can emerge.
This difference can seem meaningless but it is fundamental.
1. The use of casual data as beginning of the transforming process is similar to the logical consideration of an existing and unpredictable environmental context in which to activate a progressive process totally managed by well defined transforming rules able to interact with unexpected events.
2. The use of random parameters in the construction of formal results is an aesthetical blind search instead of following own vision identifying us as author. It defines an approach that seeks the emergent form from a process totally deprived of controls. It pursues the "death of the project", "the author's death", quoting R. Barthes, with the impossibility to recognize the author vision and identity.

The first type of approach with using different initial data is also a characteristic of my generative software: I manage the oneness of the results and the relative variations using an initial data that always changes: a number that synthesizes date and time of the beginning of the process. Then everything happens without randomness but the results, also being recognizable as belonging to my own vision, are absolutely unique and unpredictable.

**Generative Geometric figures**

We need to go over the cellular automata, that are only a particular even if extremely meaningful study case of transforming process without random. The generative geometric logics are founded upon different logical interpretations of the same geometric entities. In the generative geometry, for instance, a cube is never the same geometric event, but it depends on the logic adopted for generating it.

It could be generated defining an algorithm representing a dynamic series of solid that can go from the tetrahedron to the sphere. Or with an algorithm generating solids with two shapes existing in an orthogonal axle. Or with an algorithm representing the dynamic series from a cylinder to a triangular prism, and so on. And we could nearly define a endless series of logical interpretations of a cube that would bring to a series of solid of generative geometry that, in the construction of the generative algorithms, they totally behave in different way.

This is the base of the generative geometry.

If the Geometry is defined as "part of the mathematics that studies the space and its figures" we could define the Generative Geometry as "part of the mathematics that studies the dynamics of the spatial transformations and the progression of its figurations."

**Generative Perspective Geometry**

But Generative Geometry would be a sterile branch if there was not the perspective. It is not a case that the perspective, and its first logical form identified by Brunelleschi, has been a revolution in science. The identification of a logic perspective, or rather of a based logical structure of points of view and observed events, allowed a scientific approach based not only on deductive analysis but also to Logical Interpretations whose multiplicity is based on the points of view. The first and fundamental aspect of this "scientific innovation" has been to discover that these logical interpretations are able to acquire the infinite and "to measure it" giving an
The logical interpretations of spatial events could use different points of view and different perspective logics. These are not limited only to the perspective of Brunelleschi but they can also involve other perspective logics as the curved perspective, the anamorphic ones and the inverse perspective of Florenskij, as well as the three-dimensional representations of events with more than three dimensions. We can start from simple examples. The choice of the point of view and the logical structure of the perspective, identifying a peculiar logical interpretation of the space, can define the character of the artwork and the vision of the artist. Two examples are very eloquent. The "Flagellation of Christ" by Piero della Francesca and "the room" of Van Gogh.

"Flagellation of Christ" by Piero della Francesca and "the room" by Vincent Van Gogh. On the right a reconstruction of the room with a curved perspective from another point of view but with the sight toward the ceiling as the original image.

In both these artworks the perspective image is paradoxical, absolutely particular and hardly verifiable in the reality. Also if they both seems to be "normal" at the first sight. In the "Flagellation" the observer is very low, almost to the floor, and he looks toward the direction of the flagellated Christ. From that position he could not see in full the three figures, being these, of fact, out of an acceptable optic cone; he would see only the low part of the dresses. Instead, forcing the geometric structure of the
perspective the three figures are fully represented. The use of this point of view constructed an estranging image but geometrically "correct". And in this it reflects and renders explicit the interpretative logic of Piero. In the room of Van Gogh (C.Soddu, "The not Euclidean image", Gangemi Publ. 1986, and C.Soddu, “L'idea di spazio nelle rappresentazioni d'arte", (the space idea in art representation), in "Critica d'arte" magazine, n.16, 1988.) the perspective seems, at first sight, a normal perspective of the room seen by a standing observer. But the vertical lines converge upward. Since the observer is standing, taller than the bed and of the chair, these lines should converge downward instead. This converging is estranging because, to find again this possibility in a correct perspective image, or however in a "photographic" view, we must imply that the observer is, as he appears, more high then the objects but, at the same time, he looks upward. The whole room, therefore, would be seen with the tail of the eye while the observer (Van Gogh) is looking at the ceiling (that is not represented in the artwork) and the whole image of the room would be, in a certain sense, out of a "normal" optic cone. This posture represents, through the perspective logic, the discomfort, the character and the vision of Van Gogh. In the use of an "impossible" perspective image we can find something in common between Piero della Francesca and Van Gogh. Both have used the perspective geometry to clearly communicate a strong subjective vision of a "normal" spaces. And this has produced a spatial order strongly interpreted but, also if impossible, logically correct. It shows how the perspective science can communicate subjective visions.

1st Image, a 3D logical interpretation of a Kandinskij artwork (C.Soddu, 1987) and (2nd-3rd image) some unusual perspective images made forcing the algorithms of the perspective. When the distance change beyond its "natural" limits, if we use algorithms following the logical approach of Brunelleschi the image break itself and some elements move from one side to the other of the sheet. This happens in a different way when forcing the algorithms of curved perspective. (C.Soddu, "Not Euclidean Image", 1986. (4th image)The same approach in one of mine oil painting (C.Soddu, “Guggenheim museum NYC”, 1986) where the image is reconstructed using a spherical anamorphic logical interpretation forced
beyond the limits of this type of perspective.

The logic to represent the events identifying points of view and observed events has allowed to build different perspective logics. While the perspective of Brunelleschi and Piero della Francesca identifies an observer and an observed point, other perspectives as the cylindrical and spherical anamorphic perspective, of which I have built in 1986 the algorithmic sequences, identifies one point of observation and a linear (cylindrical) sequence or a surface (spherical) of observed points.

In these cylindrical anamorphic perspectives, representing a generated city and the interior space of a generated cathedral, the observer is in the centre of a cylinder constructed by the image wrapping the cylinder. This is the reason why the left border and the right border of the images coincide. The observer can rotate his sight looking at all possible directions. The anamorphic structure of the image answers to these different sight with a "correct" perspective image by straightening the curved lines in the perception. C.Soddu, total anamorphic perspective done with his software.

In this other total anamorphic perspective of a generated architecture the sight is oriented to the dome (and, in the other side to the floor, being a 360 degree sight. C.Soddu, software "totale" 1988.

This is the first possibility to go over the Brunelleschi perspective going over an axiomatic visual direction, opening to not Euclidean geometries. But it's possible to go ahead. The inverted perspective, identified by Florenskji in the Russian icons, inverts the direction between observer and observed point. Here, contrarily of the anamorphic perspectives, the points of view become manifold while the observed point returns to be unique. And this is indicative of the peculiar use of Russian icons: a multiplicity of people (points of view) looking at the same event, the face of the Saint. ("Perspective, a Visionary Process: The Main Generative Road for Crossing
As Florenskij argued, the Russian icons have a inverse perspective. It's possible to understand this inverse perspective because you can see, in the same time, the two ears of the Saint as we look from the inside of the head, or from the inside of a cube where the image is anamorphically projected (top fig.). The Inverse perspective is focused by Florenskij saying that we only see the eternal surface of the objects. In this case the image (button fig) is the same but the cube is inverted and we look to its external surface.

This approach using different perspective logics and the related construction of generative algorithms opened the possibility to "logically" interpret in different way the same event. The different points of view, all together, can refer to possible variations of the same logical interpretation, opening to the generation of endless possible results, endless individuals of the same species, recognizable through the same logical interpretation.

This is a way to collect our creative investigations, making them executable inside our generative software. It is possible to do that without creating a database but with generative algorithms. They, using as input different "points of view" are able to generate multiple variations. The interesting aspects of this type of generative approach are two: each result is different but each result is recognizable by the same logical interpretation, that is by the same "vision". In this way the "author" can be expressed, and the style too. This is the reason why my generative software have a lot of pages of statements. I added them step by step by following my design activity during the last 30 years.

This "change of point of view" is normally used by artists, designers and architects and it is of great utility in the creative process. As example, today I got a step of my project, of my artwork. Tomorrow I go back to my work and, to go on, I turn the sheet
on the other side and, doing that, I easily continue to draw. Making this simple
gesture, changing the point of view, I can open new possibilities and I go on
expeditious pursuing my vision and managing the complexity and the ability of my
artwork to answer to different and multiple requests. Why not to manage the same
possibility in a generative software? We can do that by using the generative
geometry for constructing our algorithms.
In the generative process, and inside the algorithms, it's possible to perform this
possibility and more. I can represent my event through a perspective representation
and then I can perform the reading of this "virtual image" as a 3D object represented
using a different point of observation. This can be performed according to my logical
interpretation, as happened in the medieval artworks by Simone Martini. He made
different representations of medieval cities. But when he represented each building,
he done it with different points of view. I discovered that it's possible to interpret
these points as belonging to a 3D line: a virtual path showing the discovery of the
medieval town. It runs from the outside to inside the city wall. In other terms Simone
Martini has used the selective variation of the points of view as way to represent the
fourth dimension in a two-dimension image. ( C.Soddu, "L'immagine non Euclidea",
"the not Euclidean Image", Gangemi Publisher 1986)

Simone Martini, tempera on panel, 1328. Looking at the different buildings it's possible to
verify that each building seems to be represented with a different perspective view. This
"interpreted" points of view create a 3D line from outdoor to inside the medieval city. We
can interpret it as a representation of the 4th dimension in the two-dimension image. In the
right image two frames of the transforming sequence of the solids following the path of
points of view. C.Soddu, "L'immagine non Euclidea", "The not Euclidean Image", Gangemi
Publisher 1986.

My opinion is that Simone Martini used, for drawing his artwork, the Generative
Geometry. And it's possible to find this type of approach in Giotto too, and in some
medieval artists living before the systematization of the perspective tools made by
Brunelleschi.
If this process is used in the creation of the space, the form of every three-
dimensional solid transforms itself in progress, assuming different results and
performing events that have characters fitting the vision of the author. Spatial orders
and characters that are logically reproducible through algorithms because the
process is repeatable.

Moving from the image as canonical perspective to a not Euclidean perspective and going back. If we read the not Euclidean perspective (first image of the sequence), as Brunelleschi perspective we can have a completely different object with rounded solids. C.Soddu, 1986.

More. We can try to read a canonical perspective as it was a curved, not Euclidean, perspective. ("The not Euclidean image", example of Balla, C.Soddu, Gangemi Pub. 1986) This generative process can produce complex solid events that reflect our spatial vision. In that case the results are rounded solids where the curved lines are strongly controlled by an intrinsic harmony, the same harmony of the previous squared solid but different fascinating. Logics are mathematically describable, therefore the construction of these generative algorithms is easily prosecutable, together with the objectives and to the characters that they intend to pursue.

Following the same approach, a reverse perspective of a cube, for example, can be read as canonical perspective assuming that it is a 5 sides prism. The increasing from 4 to 5 sides transform the solid in a generative way moving from a logical geometric interpretation to another one. ("Perspective, a Visionary Process: The Main Generative Road for Crossing Dimensions", C.Soddu, Springer)

This is the Generative Art Geometry. The hard core is constituted by the logical sliding among different representations, among different spatial dimensions. In fact, another possibility can be performed by sliding from a dimension to another. The base is moving from two dimensions to threereading a two-dimensional image as was three-dimensional and vice versa. But also managing through interpretative logics the passage from three to four dimensions, from the cube to the hypercube by reading this last event as three-dimensional.

The creative world of Generative Geometry is extremely wide, and above all it can fit the own vision. It can logically reflect our uniqueness of creative people, it is the
logical world where we can identify and develop our vision as our style.

On the left a generated baroque cathedral, together with a UFO and a car, all generated with Argenia software, C.Soddu 2013. In the right image a Generated Ship in a Japanese Sea, C.Soddu, 2014. The sea is done interpreting the image of Hokusai, 1830. The ship is the result of a generative process with a progressive geometrical transformation using the same baroque algorithms but going over the predefined limits of these algorithms. Every personal tool is made for going beyond the default limits. As it’s possible by using Generative Geometry.

References

[1] Carlo L. Ragghianti, L’arte e la critica, Vallecchi, 1951
Leonardo, 1992
[14] Paolo Alberto Rossi, La scienza nascosta, analisi delle architetture e pitture del gruppo Brunelleschi & C., catalogo della mostra tenuta a Brescia nel 1985
[15] C.L.Ragghianti, Paolo Alberto Rossi, Celestino Soddu, Il calice di Paolo Uccello uno e senza limite, in "Critica d'Arte" n. 8, pagg. 85-90,
[27] Rudolf Wittkower, Arte e architettura, 2005, Einaudi