Random Hexagons and Other Patterns Continuities

Arch. Severi Virolainen, MSc (Arch), DSc student.
School of Arts, Design and Architecture, Aalto University, Helsinki, Finland
http://architecture.aalto.fi/en/
M.A.D., Helsinki, Finland
www.mad.fi/severi/dsc
e-mail: sv@mad.fi

Abstract

This study focuses on generation of non-repeating patterns using regular polygon tessellations. Number of tessellations using regular polygons is limited. In this study I'm limiting used polygons to triangle, square and hexagon for practical reasons. Shapes are categorised by number of segments at each edge of the polygon as well if edge is symmetrical or not. Segments on edges are joined inside the polygon in all possible ways under certain rules, which gives us several different shapes for each polygon in different categories. Combining and rotating different shapes gives huge variation of different non-repeating, but continuing patterns. These patterns can be designed, random or originate from any source data. Using parametric algorithms different variations can be generated easily.

1 Introduction

Polygonal tessellations create interesting and fascinating patterns. These tessellations (tilings) are used in architecture, textiles, board games and many other applications for practical, structural and decorative purposes.

1.1 Tessellations

Using only one polygon we can create only three tessellations on a flat surface (Fig. 1.).

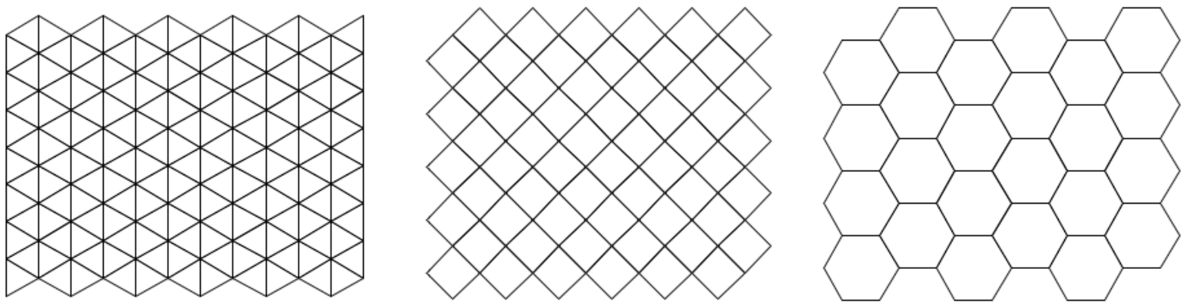


Fig. 1. Tessellations of regular polygons on a flat surface.

Using combinations of triangles, squares and hexagons we can create many more so called edge-to-edge tessellations (Fig. 2.) [1]. Some of these tessellations have a system, which allows for infinite number of almost similar tesselations.

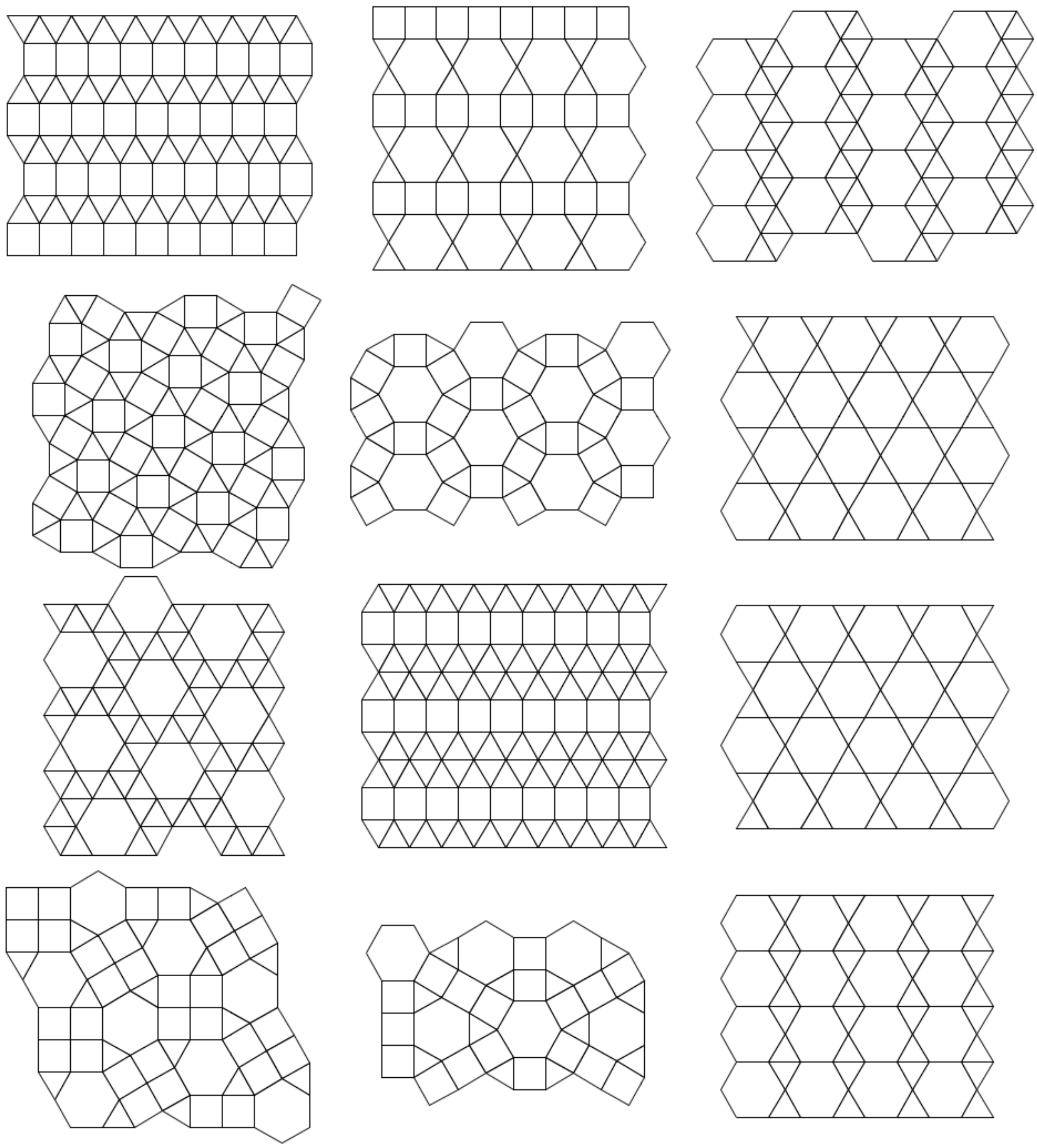


Fig. 2. Different tessellations of regular polygons on a flat surface.

There are some more tessellations if we use also octagons and dodecagons, and many more if we include curved surfaces like polyhedras, but those are excluded from this study.

1.2 Pattern continuity

By pattern continuation I mean pattern, usually a color, which continues from tile to tile creating bigger pattern.

There are different variations of this idea. A popular variation is a polygon with stripes and stripes connect different edges of the polygon at same positions at every edge. Rotating this kind of a polygon creates a pattern where stripe continues unpredictably or randomly meet itself again and closes the pattern.

One of the most successful examples is by Neil Katz [2]. One single hexagon with three stripes gives surprisingly variable stripe when rotated randomly (Fig 3.) and many interesting patterns when rotated under control (Fig. 4.).

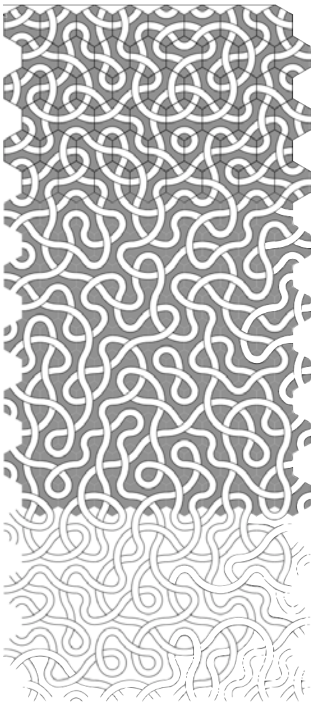


Fig. 3. One single striped hexagon used in random rotations creates interesting and non-repeating pattern of stripes.

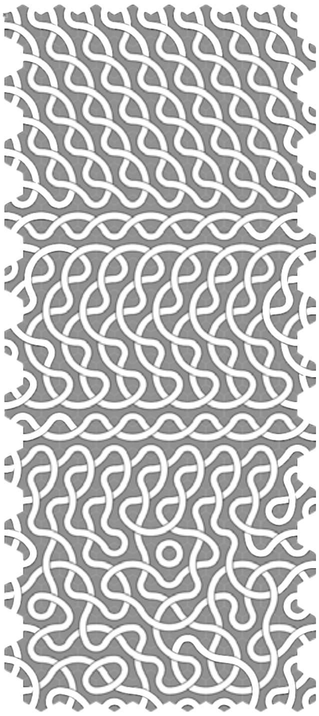


Fig. 4. One single striped hexagon used in different rotations.

The stripe has one connection on every edge. These edges can be connected in five different ways if the stripes are allowed to cross each other, and only two ways if crossing is not allowed. Naturally these five different striped hexagons can be mixed.

Using one of these tiles in certain rotation, changing rotation or random rotation we get many different patterns which seamlessly connect to each other (Fig. 5.).

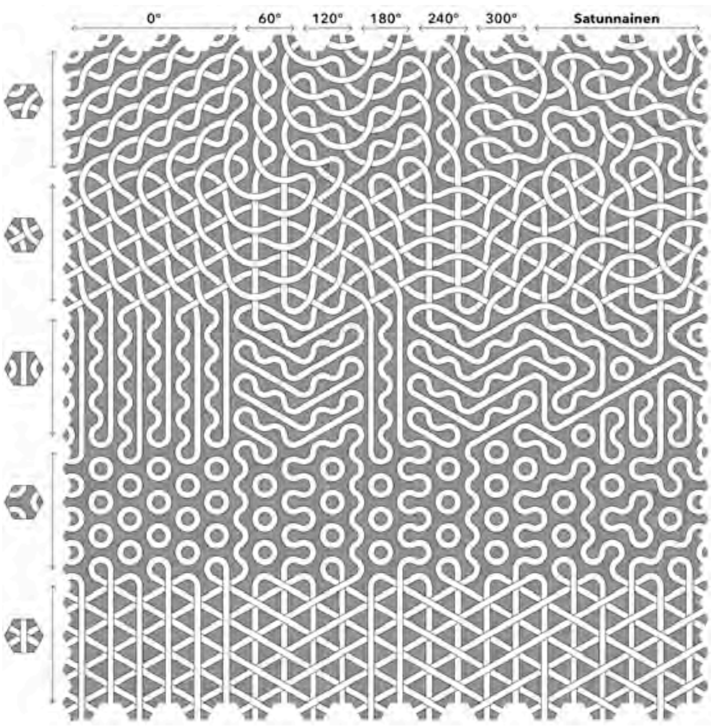


Fig. 5. Five different striped hexagons used alone in different rotations.

It is also possible to use different combinations of these striped hexagons. Pairs can be for example: two which have curved, crossing stripes, two without crossing stripes, or all five randomly combined (Fig 6.)

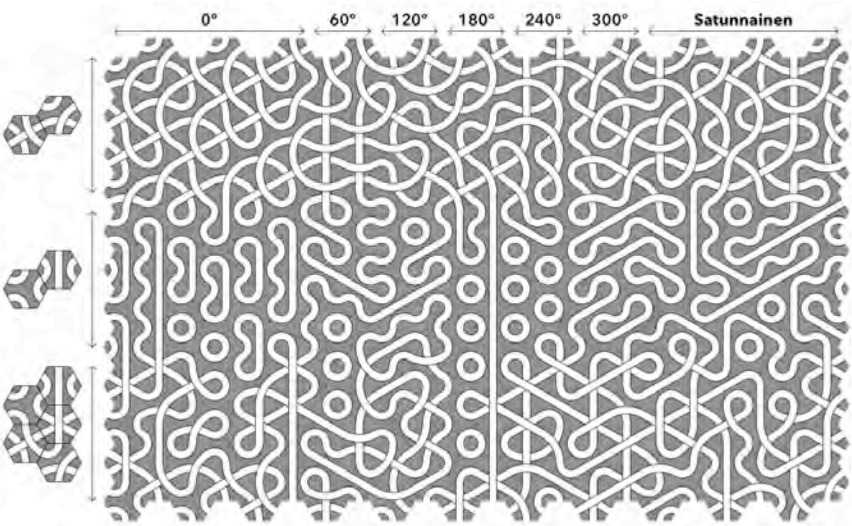


Fig. 6. Five different striped hexagons used in different combination and rotation.

It is an interesting exercise to colour the stripes, because it is extremely hard to predict where they end (Fig. 7.).

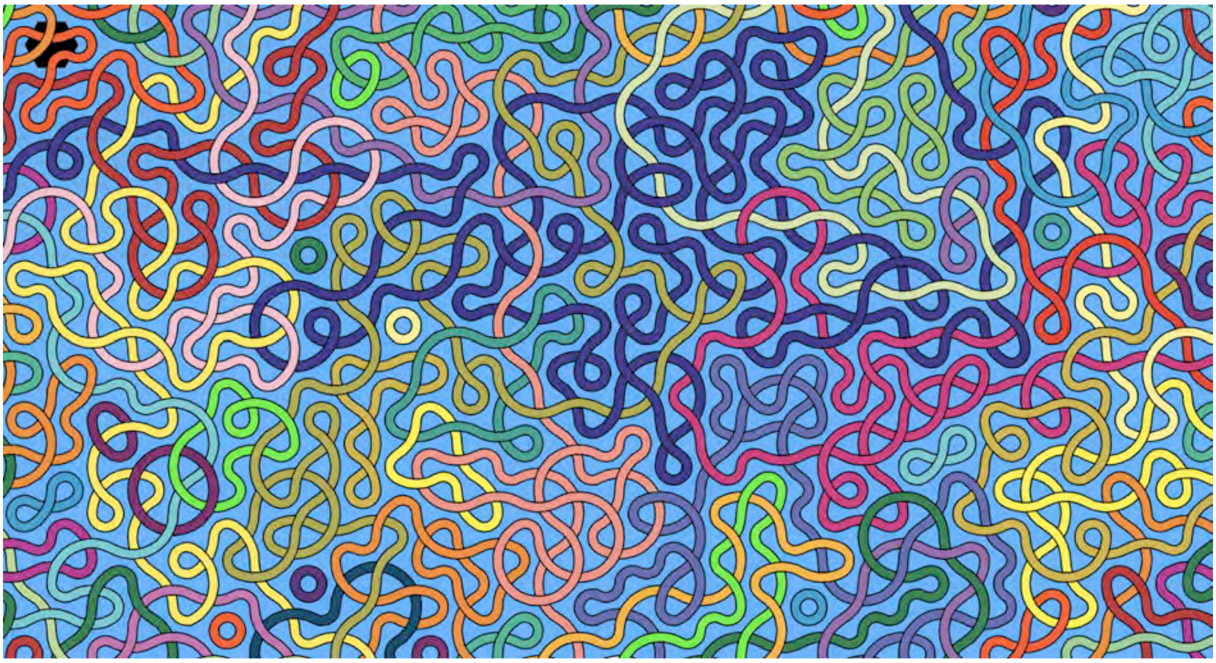


Fig. 7. Stripes hand coloured.

In this study stripes are not used. It means that stripes cannot cross over each other, so they appear more like shapes—coloured areas—than stripes.

Using those two striped hexagons where stripes do not cross we get a patterns which already have different, more graphical appearance (Fig. 8.).

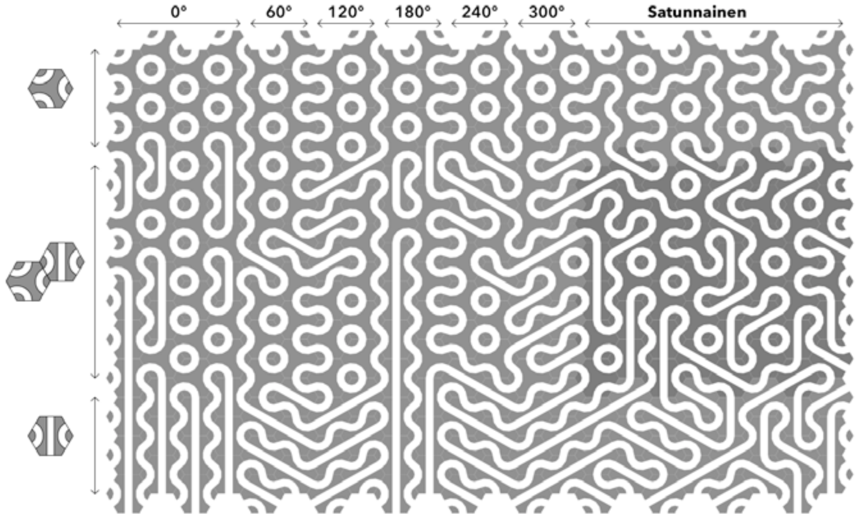


Fig. 8. (ABA 5/2) Striped hexagons without crossing stripes alone and combined. On darker area both rotation and shape are random.

1.3 Different symmetries

Edges can be symmetrical. In the example in the introduction the hexagon had symmetrical edges.

“Symmetrical edges” means that every edge of the polygon is similar and symmetrical. In the example in the introduction the hexagon had symmetrical edges. I name these edges “ABA” according to the colours of one edge. Listing the colour of every edge of a hexagon would be “ABA-ABA-ABA-ABA-ABA-ABA”, where “-“ is the corner.

“Symmetrical corner” means that the colour on both sides of the corner is the same. It is the “A-A” part, where “-“ is the corner.

Two colors are enough to show logic of four different combinations of symmetries: ABA-ABA, ABA-BAB, AB-BA and AB-AB. All symmetries are not applicable to triangle, but for square and hexagon they are. Different appearances of these are demonstrated with square (Fig. 9.).

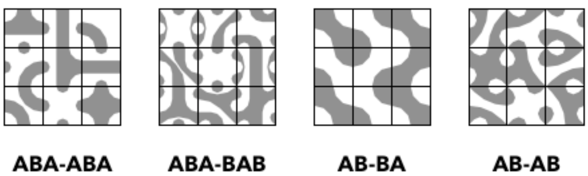


Fig. 9. Different logics of different edge and corner symmetries demonstrated with squares. A symbolises white and B grey.

ABA-ABA square has six, ABA-BAB has 28 (it has same topology as ABA hexagon in chapter 2.2.), AB-BA has only one and AB-AB has also six different shape topologies. Topologically ABA-ABA and AB-AB are same if corner is not considered as part of topology (A-A is topologically considered as -A or just A).

2 Symmetrical edges and symmetrical corners

Using symmetrical edges different shapes inside the polygon are possible. The shapes are kind of topologies. Shapes inside the polygon connecting edges could be any shape if they meet every edge similarly.

Allowing dead-ends and forking gives a lot of variation. Without dead-ends or forking triangle wouldn't have any possible shape, square only one and hexagon two (Fig. 10.).

2.1. Topologies

There are three different topologies for triangle, six for square and 28 for hexagon. If forking is allowed, but dead-ends not, left are one triangle, two squares and five hexagons (Fig. 10.).

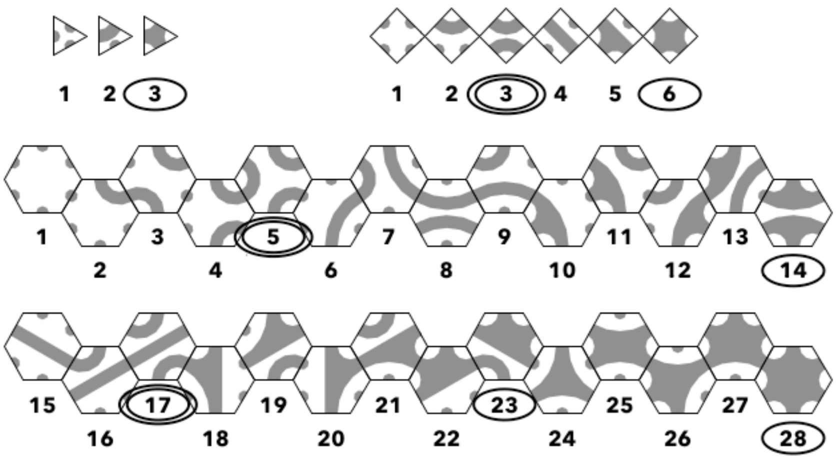


Fig. 10. Different topologies of polygons with symmetrical edges. Topologies without dead-ends are marked with small oval and ones also without forking (stripes only) with big oval.

2.2. Rotations

In this study rotation means the rotational phase of the polygon. The polygon has as many rotation phases as it has edges. In other words number of rotation phases tells how many different rotational positions polygon could have so that it stays perfectly in the grid. So triangle has three rotations, square has four and hexagon has six.

Some of the shapes are more or less symmetrical to the center of the polygon and so they have no difference in certain rotation phases. So triangles 1 and 3, squares 1 and 6 and hexagons 1 and 28 have only one appearance as they look same in every rotation phase.

Squares 3 and 4, and hexagons 5 and 24 have two appearances. Triangle 2 and hexagons 3, 8, 14, 15, 17 and 25 have three rotations. Rest of the squares have four and hexagons six rotations (Fig. 11. and 12.).

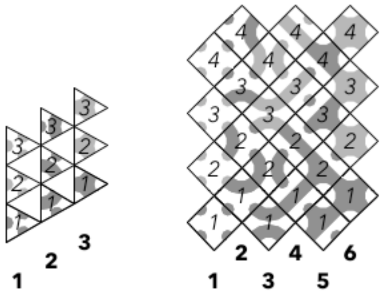


Fig. 11. Different rotations of symmetrical triangle and square. Number inside the polygons is the rotation phase.

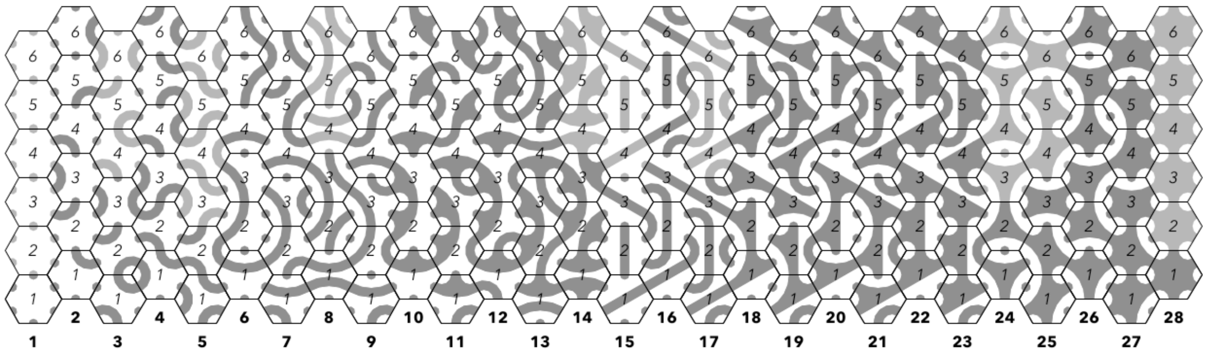
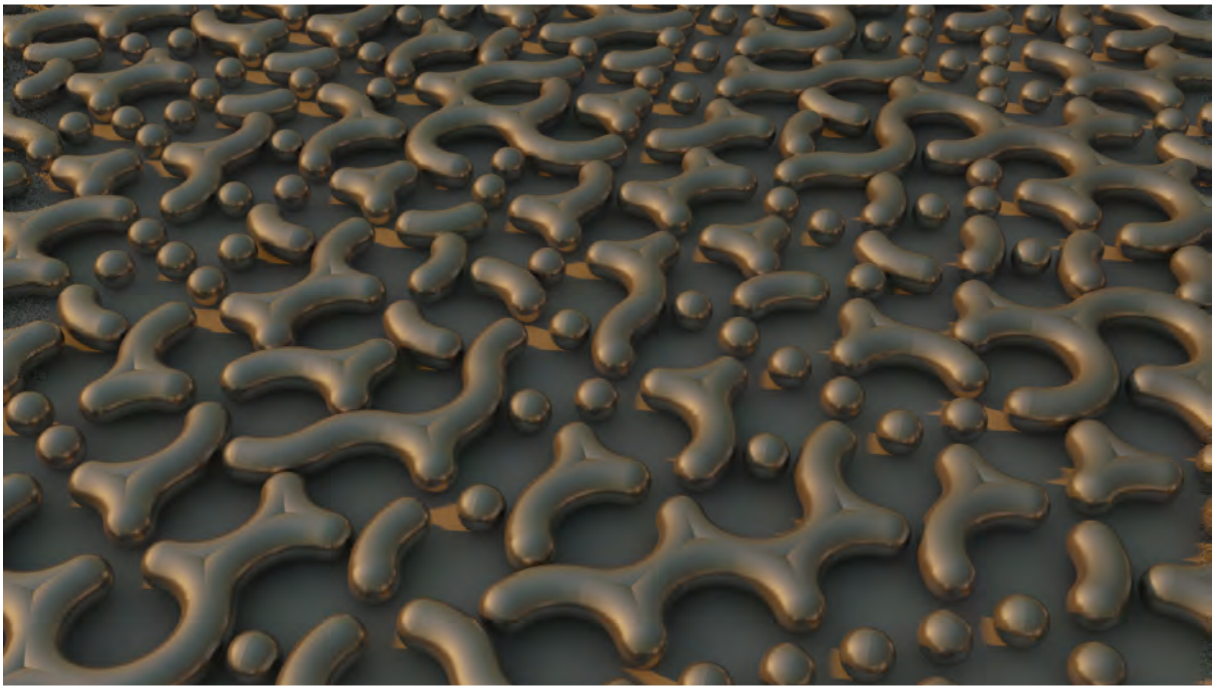


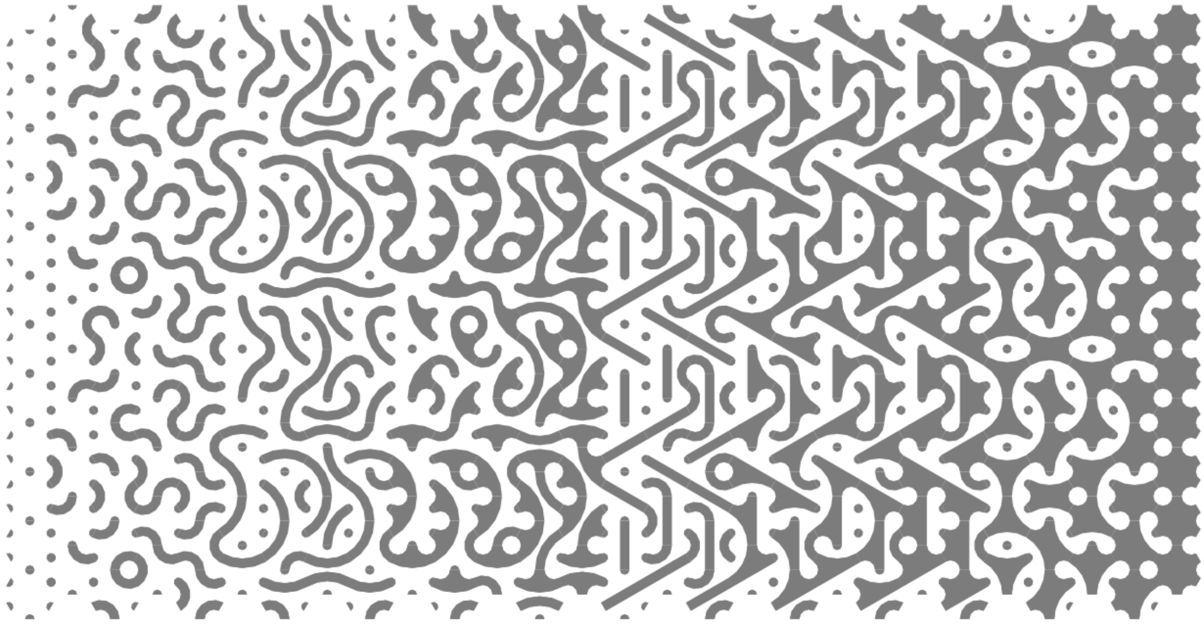
Fig. 12. Different rotations of symmetrical hexagons. Note that some topologies appear similar in different rotations. They have lighter grey pattern. Number inside the hexagons is the rotation phase.

2.3 Examples

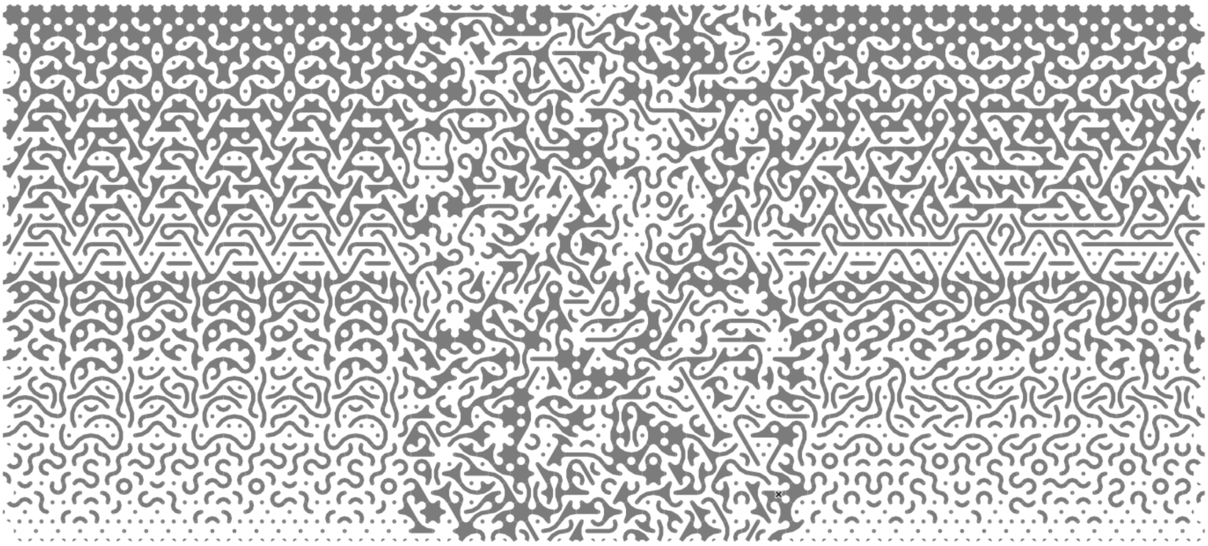
Here are examples created with polygons with symmetrical edges.



Pict. 1. (ABA 5/3) Random triangles in random rotations.

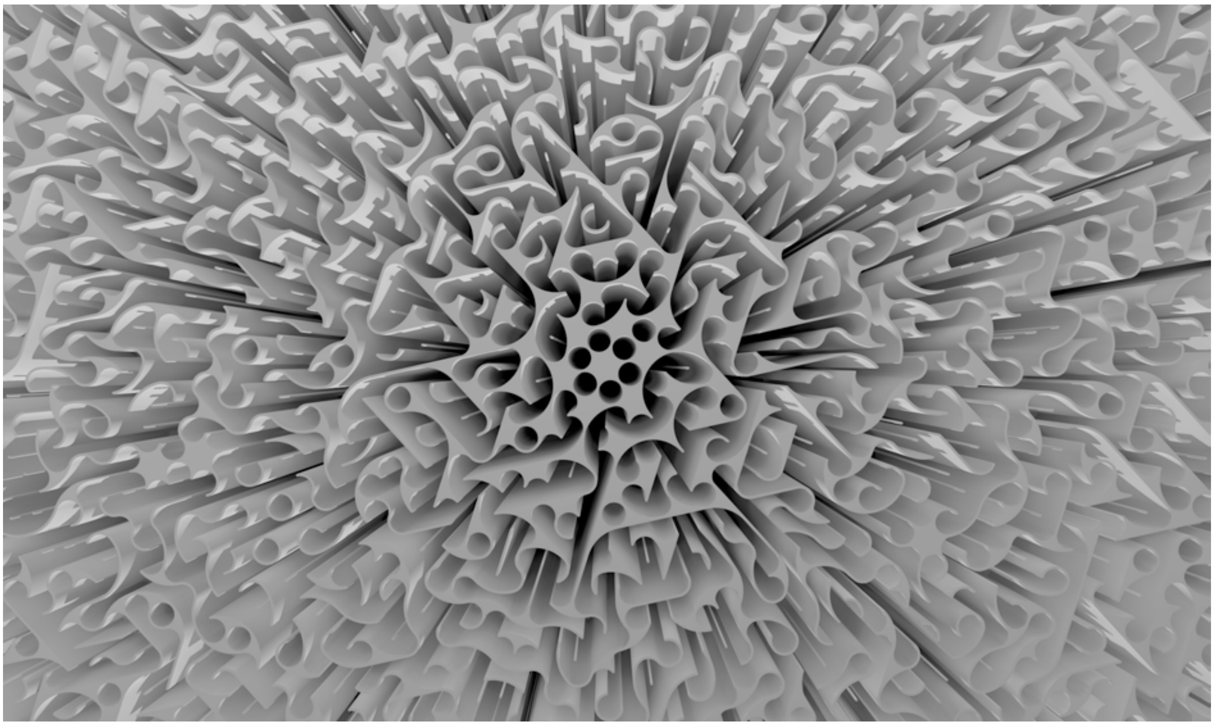


Pict. 2. (ABA 132/28) Same regular hexagonal pattern and rotation repeated as in figure 11.

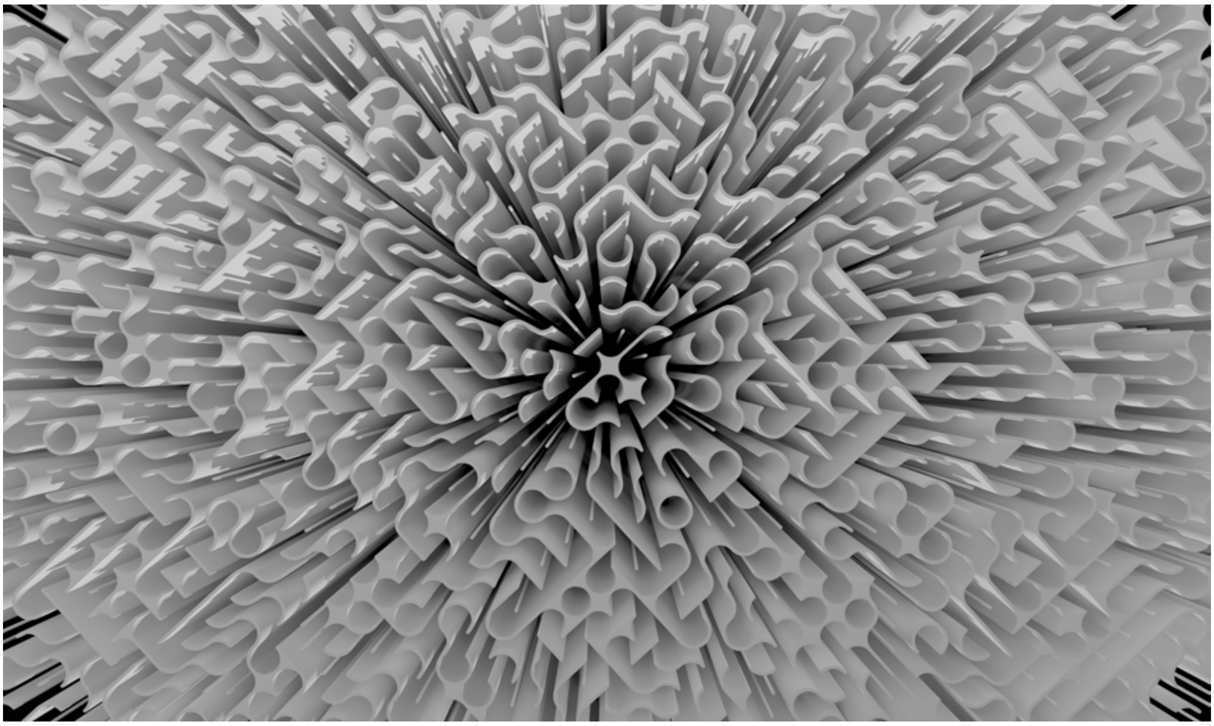


Pict. 3. (ABA 132/28) Left third is hexagonal patterns 1-28 from bottom to top in rotations 1-6 (from left to right). Middle third is hexagonal patterns 1-28 randomly in random rotation. Right third is hexagonal patterns 1-28 from bottom to top, but rotation is random. This is to show how order can be controlled and surprisingly left and right thirds appear at first sight almost similar.

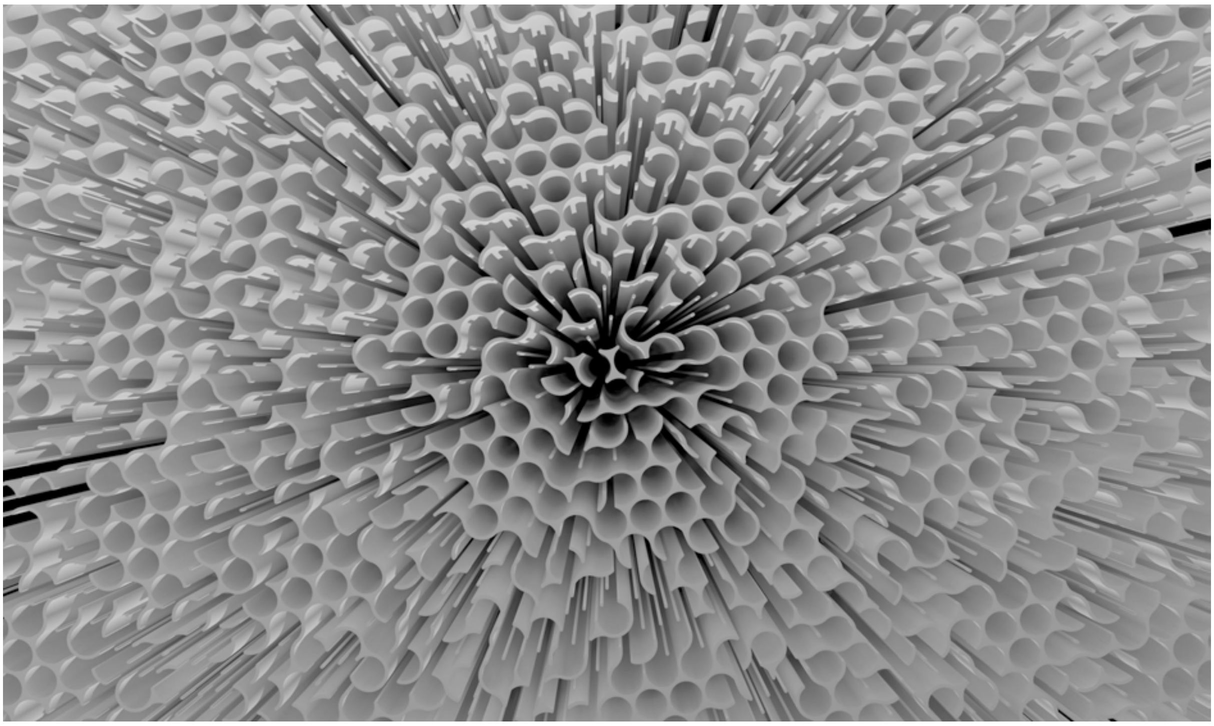
Pict. 4. (ABA 132/28) 3D labyrinth created with hexagons.



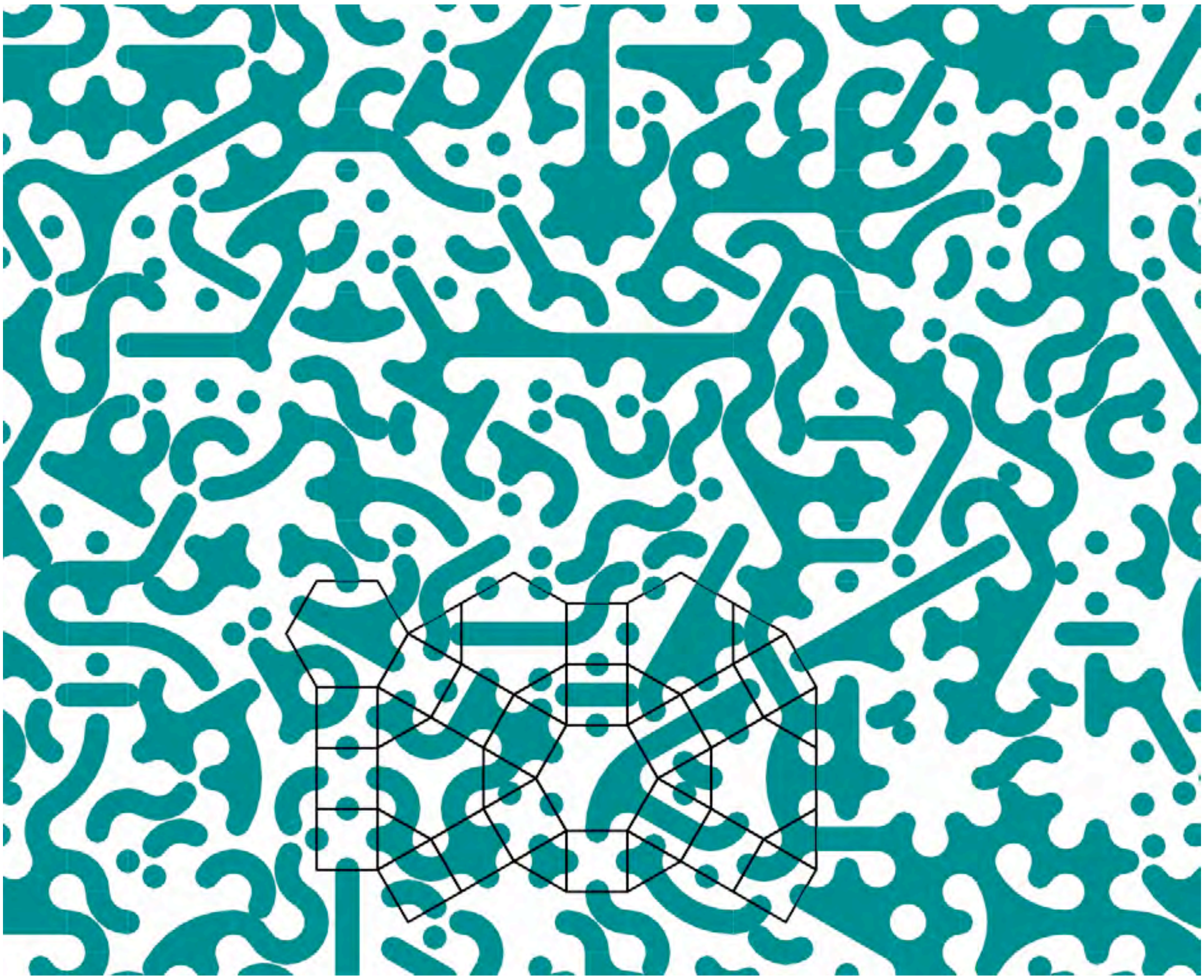
Pict. 5. (ABA 132/28) 3D labyrinth created with hexagons.



Pict. 5. (ABA 14/6) 3D labyrinth created with squares.



Pict. 5. (ABA 5/3) 3D labyrinth created with triangles.



Pict. 6. (ABA) Total chaos is tightly organized with tessellation combining different polygons. All patterns and rotations are random.

3. ABCBA

“ABCBA” type of edge behaves like “ABA” in two layers. The inner or top shape just must fit inside outer or bottom shape. I call outer or bottom shape as master shape and inner or top shape as sub shape. We get 255 different ABCBA hexagons, 19 squares and six triangles (Fig. 13.).

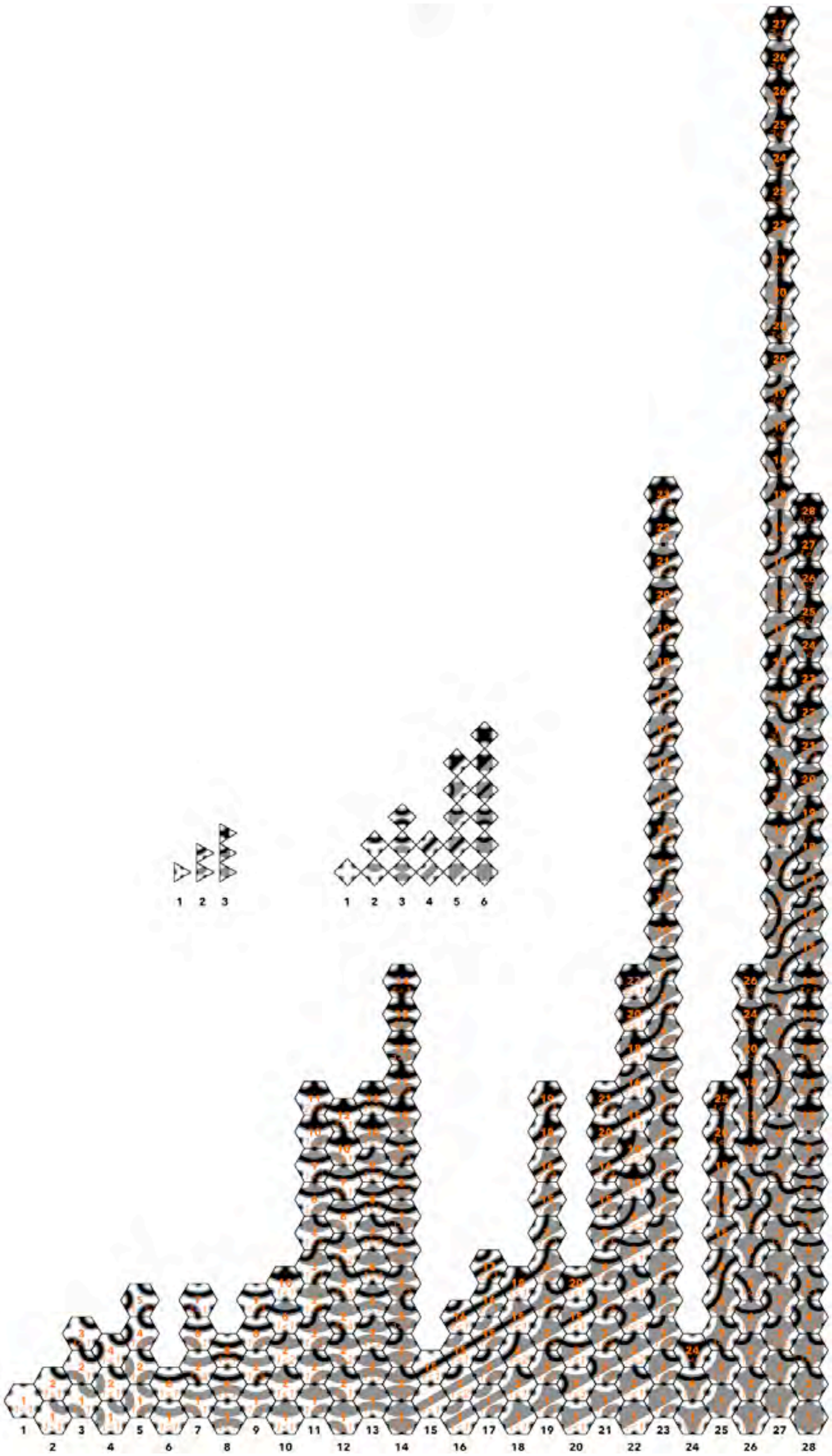


Fig. 13. Schedule of different ABCBA type polygons. Grouped by master shape. Inside hexagons is marked number and rotation of sub shape. 6 triangles, 19 squares and 255 hexagons.

Patterns created with “ABCBA” edges are naturally one level deeper and thus more interesting (Fig. 14., 15., 16.)

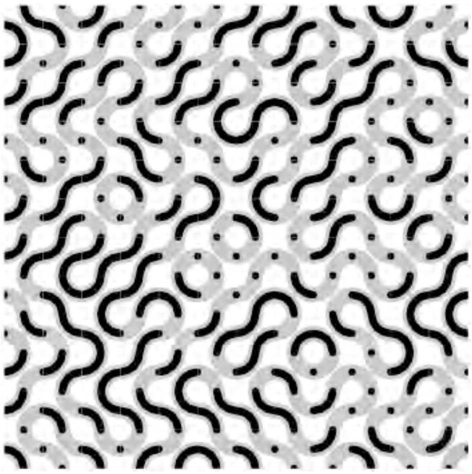


Fig.14. ABCBA (8/3/4) squares number 3.

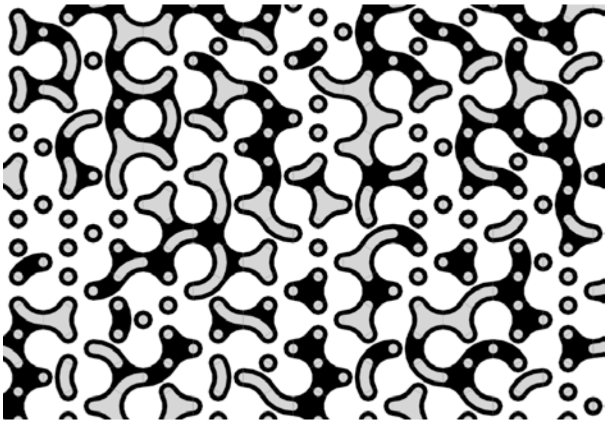


Fig.15. ABCBA (9/3/3) triangles.

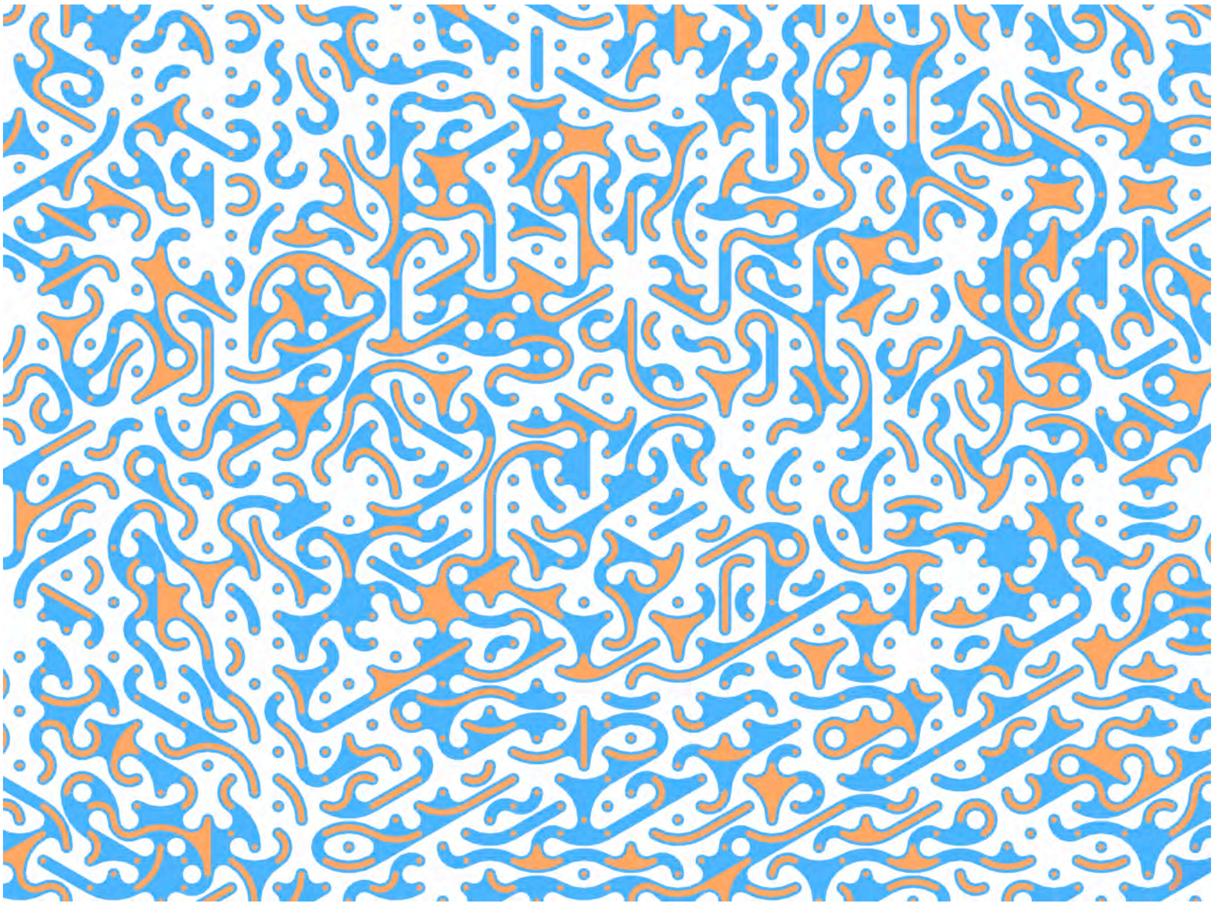
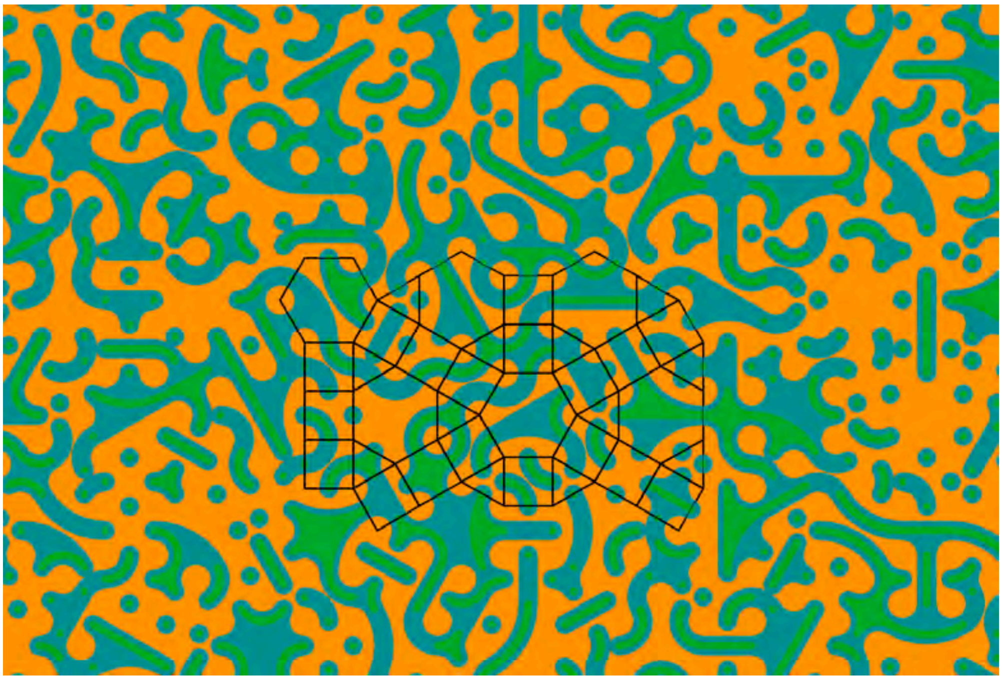
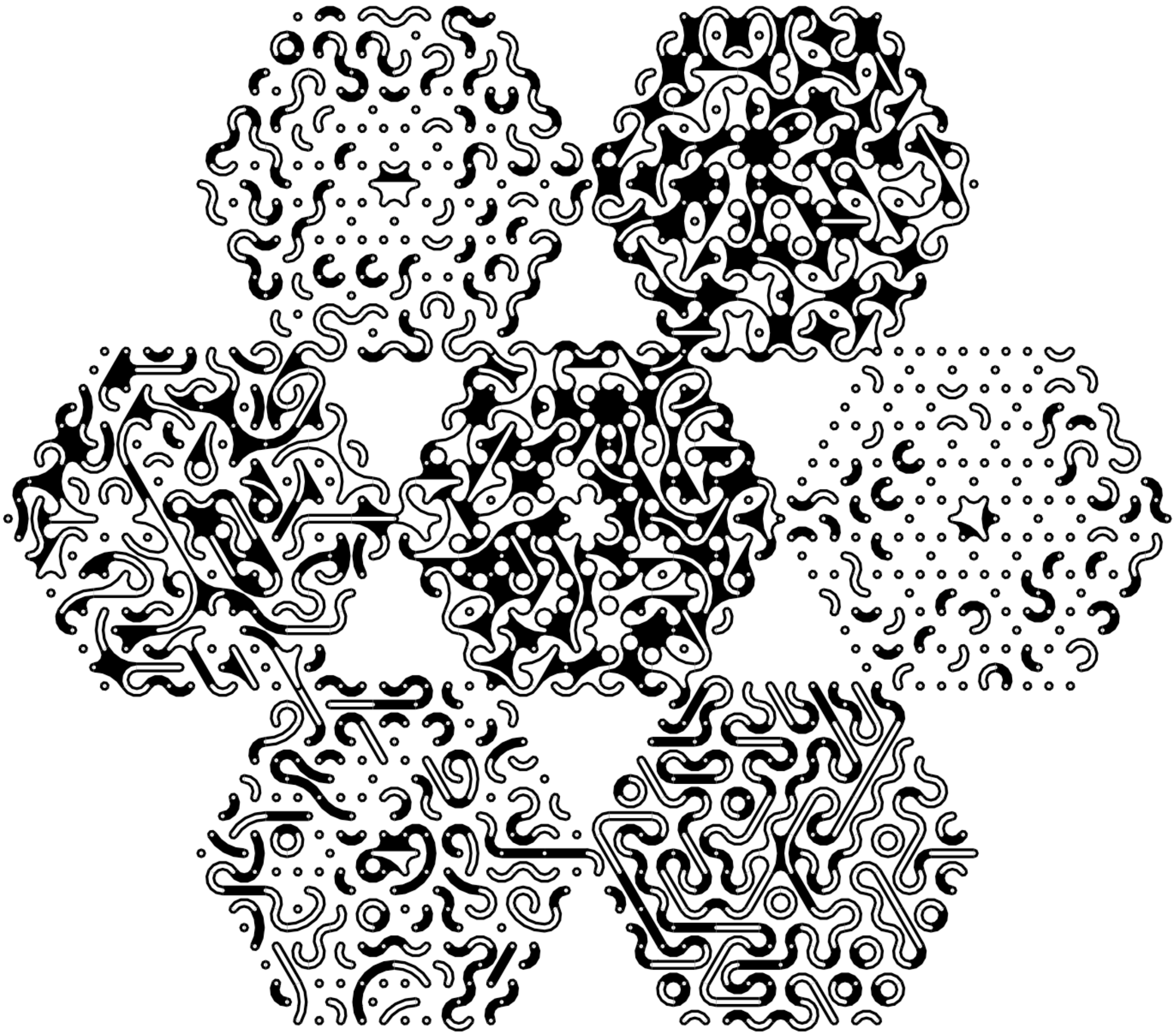


Fig.16. Random ABCBA hexagons. Upper part of the image they are randomly rotated and at lower part they are all at rotation 1 (angle 0).



Pict. 7. Total tightly controlled chaos with different ABCBA polygons.



Pict.8. ABCBA hexagons. All seven tessellations have different parameters for group of random hexagon shapes. All centre hexagons are 28 except the absolute centre which is 1.

3 Future work: Asymmetrical edges and asymmetrical corner

Here is a small glimpse to the next steps in this research.

Asymmetrical edges mean that single edge of the polygon is asymmetrical. This means that edge of neighbouring polygon must be mirrored for the pattern to continue in the next polygon. Simplest asymmetrical edge is “AB” and the edge of neighbouring polygon must be “BA”.

Also corner can be symmetrical or asymmetrical. In chapter 2 asymmetrical corners were not covered. Asymmetrical corner means that colour on both side of the corner are different, such as “B-A”.

Chapter 1.3 shows different combinations of symmetrical and asymmetrical edges and corners.

In this chapter argumentation is missing, as this work is still to be done.

3.1. AC-BC hexagon

For hexagonal tiling of asymmetric corner three colours are needed. For triangles and squares two colours would be enough. The need is similar to chessboard. For hexagonal chessboard three colours are needed (Fig. 16.).

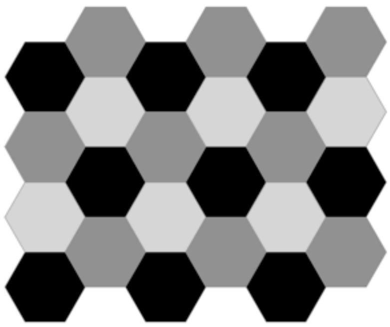


Fig. 16. Hexagonal “chessboard” pattern demand three colours.

Similarly AC-BC hexagons are needed in three “colours”. So actually there are also variations BA-CA and CB-AB. These variations are colour phases (Fig. 17.).

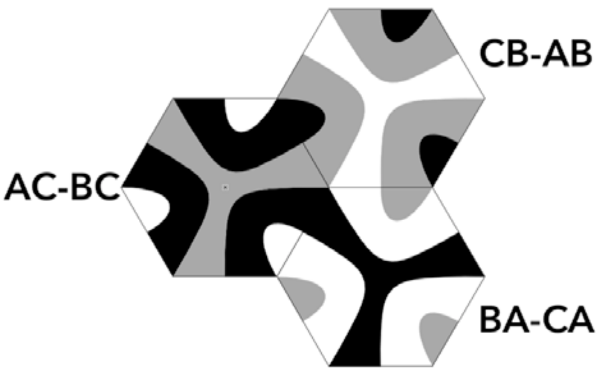


Fig. 17. Different colour phases of AC-BC are BA-CA and CB-AB. A symbolises white, B grey and C black.

To make this colouring system work, only three shapes (topologies) are possible. It is important to note that AC-BC hexagons cannot be mirrored or pattern (colours) will not continue from polygon to polygon. Those shapes are named here as “I”, “Y” and “*” (star) according to their dominant appearance (Fig. 18.).

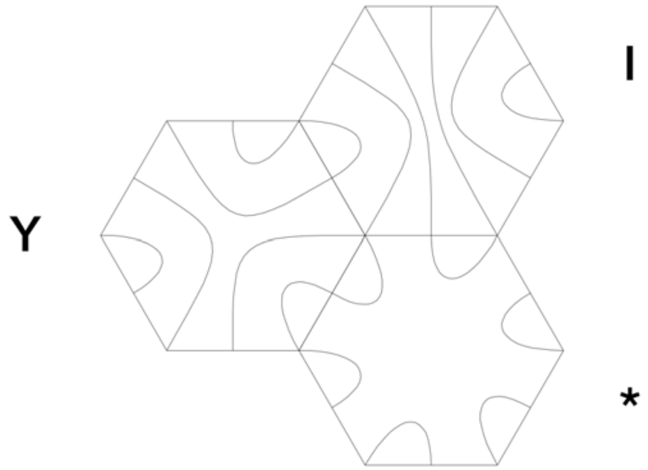


Fig. 18. AC-BC hexagon shapes are named here as "I", "Y" and "" (star) according to their looks.*

Star has only one rotation, Y has two and I has three. Rotations can be controlled or random (Fig. 19.).

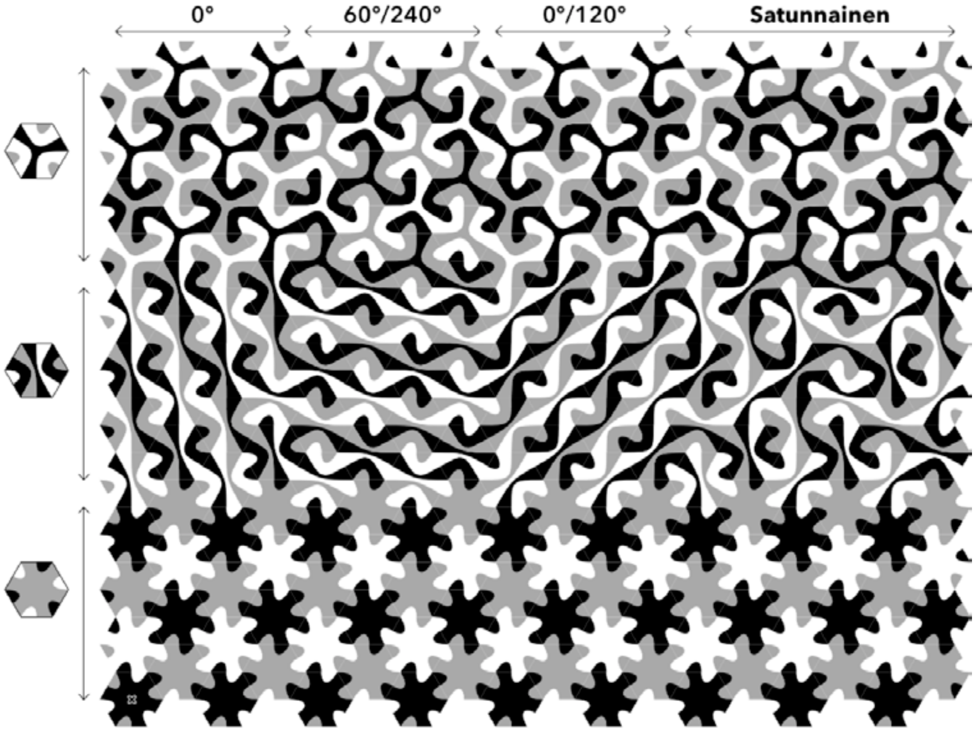


Fig. 19. AC-BC hexagons in different rotations (Satunnainen means random).

If they are also randomly mixed resulting pattern gets more interesting. Random selection can be either by shape or taking also rotation in considerations. If shapes appear equally many times Star will be dominating, because it looks the same in every rotation. If shapes are selected taking in account also their rotation the result is less Starry (Fig. 20.).

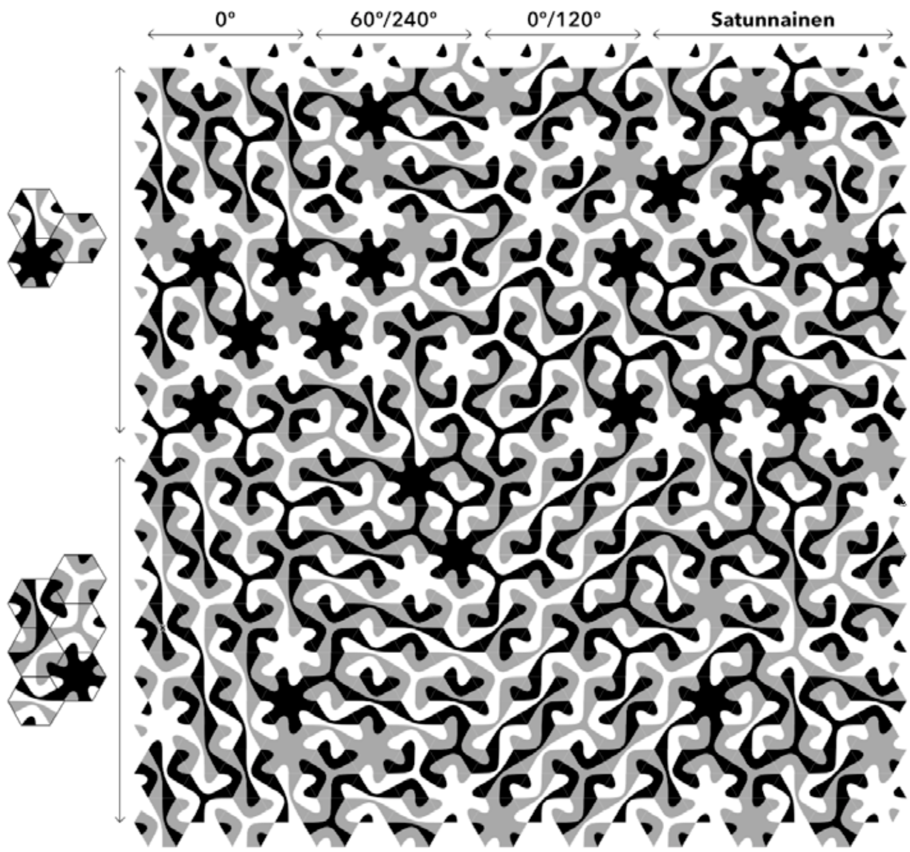


Fig. 20. Hexagons selected randomly. At upper part one third is each shape. At lower part every rotation has one sixth. Stars are more dominating at the upper part.

3.2. ABC-ACB and ABCA-BACB hexagons

All colour systems are not possible, but surprisingly many are. For example ABC-ACB hexagon gives similar, but different patterns as AC-BC hexagon (Fig. 21.).

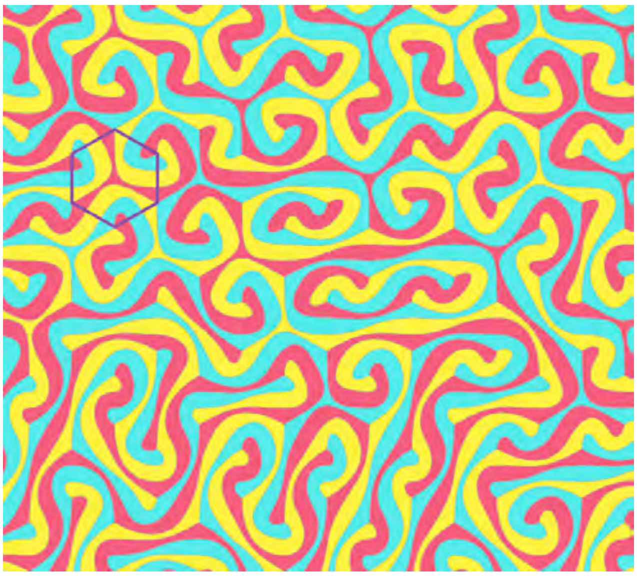


Fig. 21. ABC-ACB hexagons.

Also ABCA-BACB hexagon is possible (Fig. 22.). As this is future work, the logic and limitations of all different possibilities are not yet known.

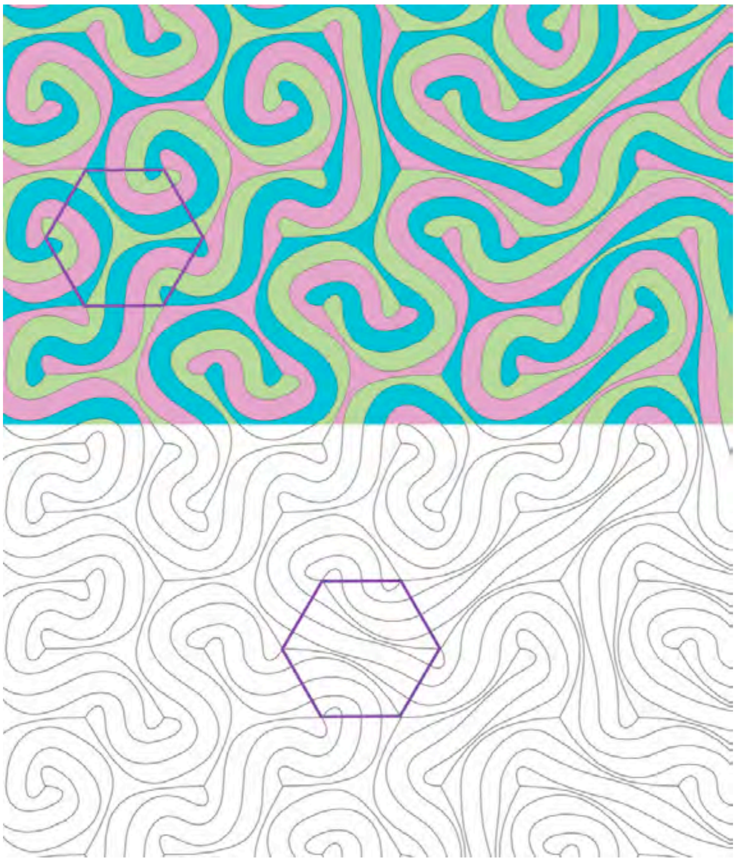
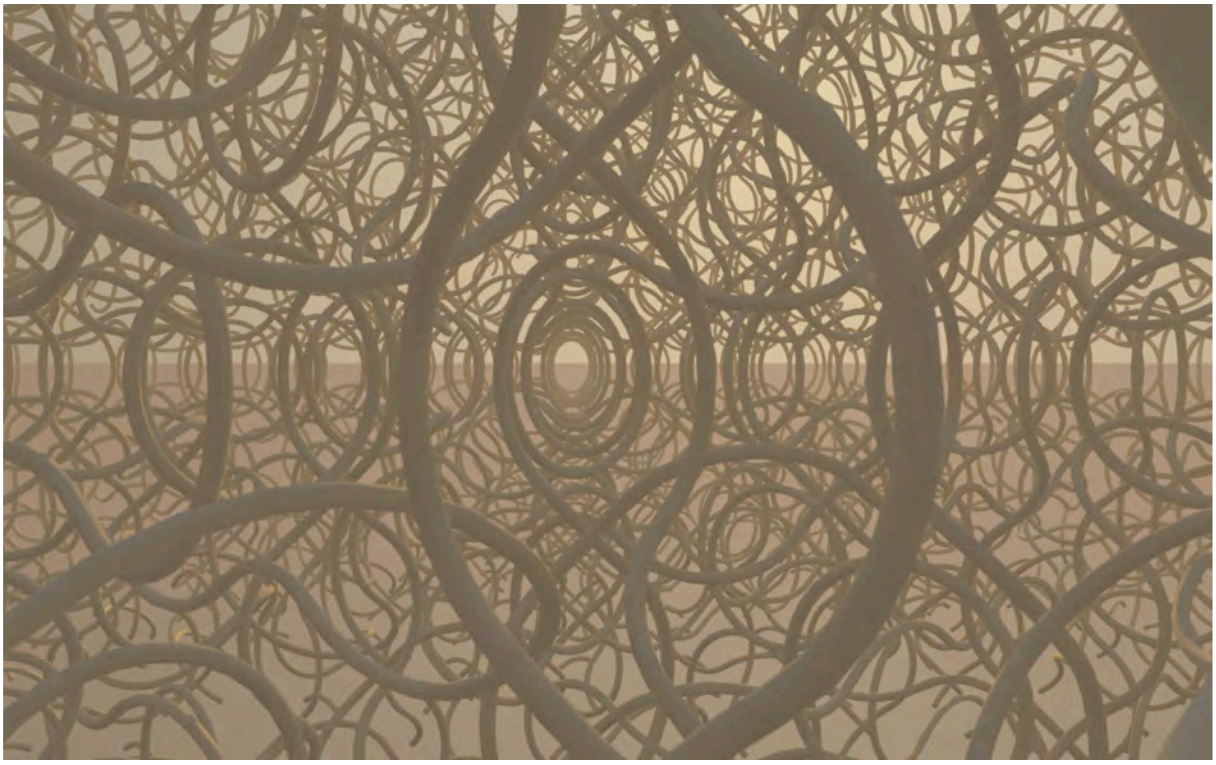


Fig. 22. ABCA-BACB hexagons.

3.3. 3D

After solving the logics of two dimensions, next is applying them, if possible in three dimensions.



Pict. 9. One 3D application “ABA” without forking or dead-ends.

References

1. Grünbaum, B., Shephard, G. C.: Tilings and patterns, New York: W. H. Freeman and Company, pp 58-71 (1987)

2. Katz, N., Peters, B, Peters, T: Inside Smartgeometry: Expanding the Architectural Possibilities of Computational Design, West Sussex: John Wiley & Sons Ltd, pp 80-81 (2013)