FROM PHYSICAL AND MENTAL OPERATIONS TO GENERATIVE PROCESSES

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Abstract:
Architecture deals with matter, and, from the first attempts to make shelters to the most sophisticated techniques of our time, a number of physical operations and transformations are carried out in order to transform materials into forms able to accommodate human life.
Progressively, architecture has become a more intellectual activity implying mental, or conceptual, operations, in great part because the scale of buildings implied drawing a project before it was built. Without ignoring the necessities of construction, those operations and transformations have acquired their autonomy, have sometimes become a game, or an art, for itself.
These physical and mental operations resort mainly to topology and geometry, or one could say that they contributed to develop those sciences that constitute our essential knowledge on space and forms.
Generative processes differ from the other ways man has had to fabricate and even conceive forms : they work generally in the digital world (even if the forms obtained may be built), which differs from the physical world, and they function more like the ways nature itself makes forms (growth, recursivity, etc.). But generative processes that tend to generate forms use, as traditional ways man has had to make forms, topological and geometrical operations/transformations ; some processes (IFS) even allow us to define a form *only* by a set of geometrical transformations, which are iterated. That must lead us to question our understanding of space and forms.
This paper focuses on some topographical and geometrical operations, and their physical and conceptual manifestations in the history of architecture and more generally of man-made forms. It discusses their use in generative processes, illustrated through works by the author.

In his paper titled « Gencities and and visionary Worlds » [1], Celestino Soddu invited us to consider a pyramid as an example of an investigation on species, in order to explain the logical structure of creation. Pursuing this lead, we'll explore the physical and mental operations involved in making forms, and the generative approach to forms, through the same example of the pyramid.
This paper also attempts to make a link between different concerns, issues and experiments exposed in previous papers.

Making forms

To fulfill their various needs, human beings resort to forms as they encounter them, or transform matter to make forms. There are no pyramids (to my knowing) in the natural world of forms. Some shells are in the shape of cones, some stones may be in the shape of regular or semi-regular polyhedra, but we don't encounter pyramids as such.

The way in which we are able to make a pyramid depends first upon the type, or state, of matter we use. With some solid matter like stone we may, as Celestino Soddu suggests, « cut a series of pieces from a cube » (supposing that this matter presents itself initially as a cube) « through plain cuts », though that technique would better fit a semi-solid matter like clay. If a sculptor hasn't got a saw able to cut stone, he would rather cut chips out of a block of stone until he gets the rough shape of a pyramid

(having probably drawn some lines to guide its work), and then sand and polish it to get plane surfaces.
With clay or some other malleable matter, one can model a pyramid by pressing with his hands on the mass, and continuously transforming the shape of the volume of clay into a pyramid.
But one can also find a mould in the shape of a pyramid, pack the clay (or even better pour plaster, which will become solid) in it, and remove the mould to get a pyramid. It's the technique of the mud pie, and supposing some child has got a bucket in such a shape, and wet sand, he can mimic his own surroundings of Cairo on the beach.
A lot of solid forms that we use begin with a semi-solid, or even fluid, matter, easily transformed into the wanted shape, and then processed in order to become solid (or processed to become fluid, and then restored to its solid state) : adobe, mud, is easily modelled and then either cooked or let to dessicate in the sun ; glass, or metal, is heated up to fluidify it, modelled, and then returns to its solid state when cooled.
By the way many solid forms that constitute our geological environment have acquired their form in fluid or semi-fluid state. Mountains are generally the result of such deformations. Though mountains are not pyramids (as Benoît Mandelbrot is fond to say), the pyramid is nevertheless a common abstraction of such shapes.

The mould that can be used to make a pyramid introduces the issue of what we attempt in making a pyramid : a filled volume, or an hollow one ; the volume, or the surface (or pseudo-surface) of a pyramid. All techniques above stay in 3D : volumes are cut, or transformed ; but usually to make a mould, one takes and cuts some pseudo-surface (a metal sheet, for instance) and folds it.
Such a hollow pyramid can also be easily made by sticking four poles in the ground, linking them at the top, and putting cloth, or any other sheet on them.
Pyramids are not all filled, like those of the Pharaohs, which where not destined to living human beings and, for that reason, may be discussed as actually belonging to architecture. Louvre Pei's Pyramid is only the quasi-surface of a pyramid made of sheets of glass (and without a base, even). Living human beings need holed volumes to inhabit.

An important issue in considering how to build a pyramid is its size. Egyptians, even if they had the idea of making a pyramid by cutting a great stone or moulding clay, couldn't have built their huge pyramids that way, because of the size they wanted to give them. Architecture leads often to assemble small parts to do a big form. So the great pyramids were built by stacking stones (there is a theory that pretends that those « stones » were themselves moulded), which was a difficult task enough (but possible). The pyramid is the sublime abstraction of the heap.
A young French artist, Simon Boudvin [2], has explored the concept of the heap, especially remarking that any architecture is in a way a heap of matter, and that this matter has had to be taken somewhere, leaving a hole. One might also remember that general Napoléon Bonaparte (not yet just « Napoléon »), reflecting upon the Egyptian Pyramids, not only famously said : « Soldiers, from the top of those Pyramids, forty centuries are looking at you !» (or something like that), but estimated that the stones present in one of those Pyramids would be enough to build a wall all around the borders of France...

Thinking forms

Through making forms, man has discovered laws of space and forms, which are expressed in those branches of mathematics that are geometry and topology.

It is maybe extreme to say that geometry derives from human actions (some scientists or philosophers may disagree), but it is nevertheless an opinion that may be sustained. Artists, and especially architects, had to abstract his making of forms because of the size of the forms they intended to make, and also because of the necessity to explain their design to other people who would actually build it.
Representing forms has been an important step towards thinking forms, because it generally implies, at least, a scaling of forms, either by drawing them on a sheet of paper, or by making models : in both cases, those representations need be not too large to be handled. Drawings go further into abstraction, because they not only scale forms, but they imply projections, i. e., reducing from 3D to 2D, either by orthogonal and other parallel projections, or by perspective.
Geometry sees figures as sets of points, those points being linked by straight lines, those lines being again linked by straight planes. For example, geometry considers the pyramid essentially as a set of five points, the vertices of the pyramid : four points in a plane that are the vertices, or corners, of a

square, and a fifth vertex, which is the centre of the square, lifted up at some level. What we call « square » is indifferently its set of vertices, its perimeter or its interior, though those three instances are dimensionally distinct, because once you get the four points, there is no ambiguity about what you can do with them in the context of geometry. So geometry can say that the square is defined by its type of symmetry (four axes of symmetry, its bisectors and diagonals), and even that this type of symmetry leads only to the square ; or that the pyramid is defined by its type of symmetry (four planes of symmetry defined by those of the square), and even that this type of symmetry leads only to the pyramid.

There are many ways of exploring forms, of *thinking* forms, one of which is to wonder about folding patterns. Paper or cardboard is a good instance of pseudo-surface, and by cutting, folding, and pasting it, we can imagine abstract operations on surfaces. One can obtain a pyramid by drawing four equal (isosceles, or even equilateral) triangles linked to each other by their side, cutting the pattern, folding it in order to link and glue the two free sides, and by putting it on a plane surface :

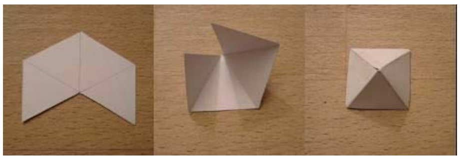

Fig. 1 Folding a pyramid (var. 1)

Or one can obtain a pyramid by drawing equal triangles on the sides of a square, folding and glueing the pattern accordingly :

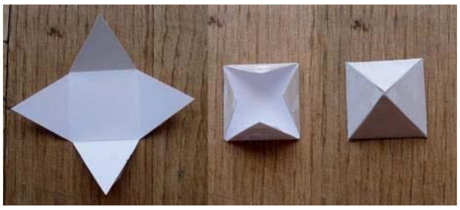

Fig. 2 Folding a pyramid (var. 2)

This operation, which links 2D (the surface symbolized by the paper) to 3D (the volume bounded by the surface), lets us think what could be a 4D-pyramid. Actually this is a very simple polytope : as a pyramid is constituted by a square and four triangles, a hyper-pyramid is constituted by a cube and six pyramids, one has only to imagine linking the 6 faces of a cube to a 9^{th} vertex (in the fourth dimension). A 3D projection of that is a cube with 6 adjacent pyramids inside ;other projections (or if the hyper-pyramid rotates in the fourth dimension) would show the 9^{th} vertex going our of the cube.

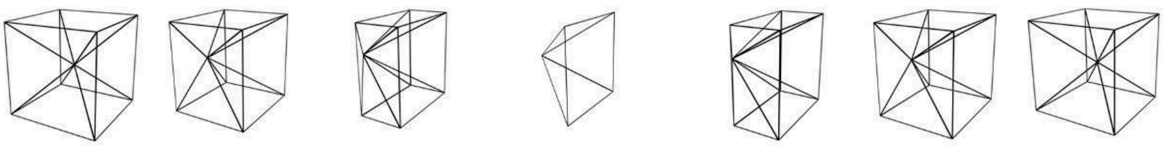

Fig. 3 Rotation in 4D-space of a 4D-pyramid

That reminds us that a cube may be built with 6 pyramids, each one with its height being half the side of its base :

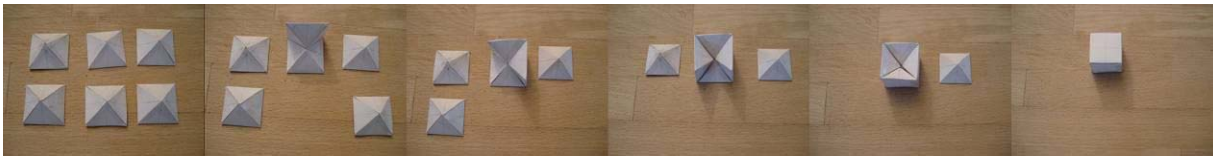

Fig. 4 Six pyramids filling a cube

But we can also put the pyramids outside a cube, rather than inside. We know that by surrounding a square with the four triangles that are defined by its diagonals, we get a square, the area of which is twice the area of the initial square. Sadly, and because the laws of 3D space are not the same than those of the plane, we don't get a cube by surrounding a cube with the pyramids defined by its diagonals. But we obtain an interesting polyhedron known as a rhombic dodecahedron (and its volume is twice the volume of the cube) :

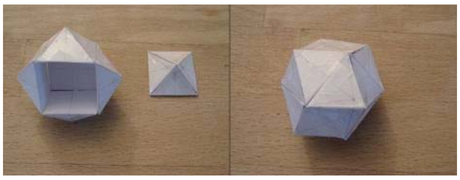

Fig. 5 Pyramids around a cube make a rhombic dodecahedron

Then one can wonder how to fill a pyramid with pyramids. A pyramid contains five pyramids half its size, but there remains four holes ; if the triangles of the pyramid are equilateral, those holes are regular tetrahedra :

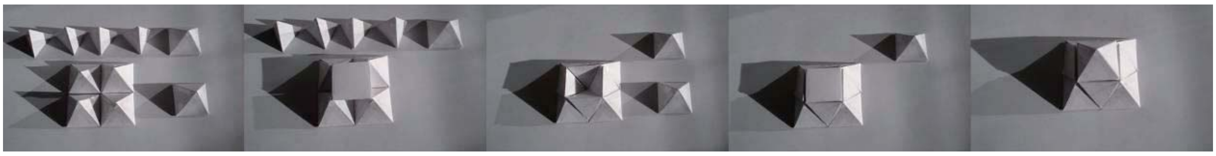

Fig. 5 Making a pyramid with 6 pyramids and 4 tetrahedra

This reminds us that a regular pyramid is half an octahedron :

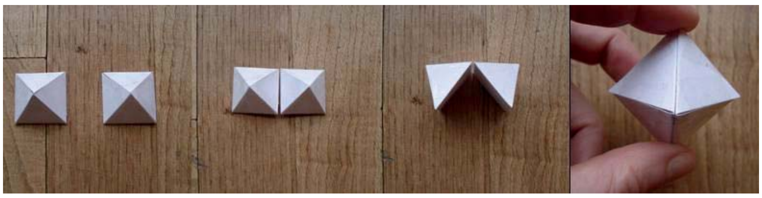

Fig. 6 Making an octahedron with two pyramids

To finish this exploration of folding paper, let's mention the art of origami. There are many ways to obtain a pyramid through origami, which let you get something like that :

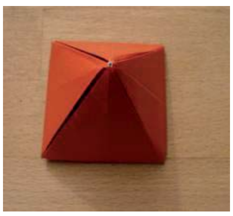

Fig.7 A pyramid in origami

Generating forms

Generating forms means considering forms in some different way, not by describing them by all their geometrical characteristics, but as the result of applying rules : there is a shift in the attitude of the designer, who doesn't (and doesn't want to) know exactly what he will obtain, though he has total freedom to elaborate rules of his choice.
That links generative art to how natural forms are made : no superior intelligence has decided the exact form of such or such mountain, or of such or such tree, or even of such or such animal. Natural forms are the (always provisional) result of processes, applying rules related to physical forces, and/or biological patterns. We can describe, analyse, and measure them at some given time, but they cannot be really understood without the fact that they develop themselves in time.

Physical and spatial laws

How can a pyramid appear « by itself », applying only physical laws ?
José Bico and colleagues of the research laboratory Physique et Mécanique des Milieux Hétérogènes (MPPH) at the Ecole Supérieure de Physique et de Chimie Industrielles de la Ville de Paris (ESCPI) have developed amazing experiments they call « capillary origami ». It consists in putting a water drop on a flexible sheet, previously cut along a pattern. Pictures and videos on his website [3] show how a tetrahedron or a cube appears miraculously only through laws of capillarity. No doubt that a pyramid could also be obtained this way.
A way to obtain a pyramid by applying only laws of gravity is to pour sand onto a square, permitting sand to freely fall from the edges. You may obtain a rough pyramid depending on the quality of the sand and the size of the square :

Fig. 8 A pyramid obtained by pouring sand on a square

The slope of the pyramidal heap depends on the sand (around 30°).
I developed the idea of distance maps in a previous paper [4], and linked it to sand heaps. A pyramid appears when the centres of the distance map form the perimeter of a square. The slope of the pyramid depends on the ratio you decide to affect to the level of grey in relation to the distance.

Fig. 9 Distance map of a square and its translation into a mesh

Cellular automata

Pyramids occur with 3D CA with the following rules : starting with one cell, a cell appears if it has neighbours on one of its sides or corners or just below (a 3D cell has 26 neighbours). That rule mimics one way of physically building a pyramid :

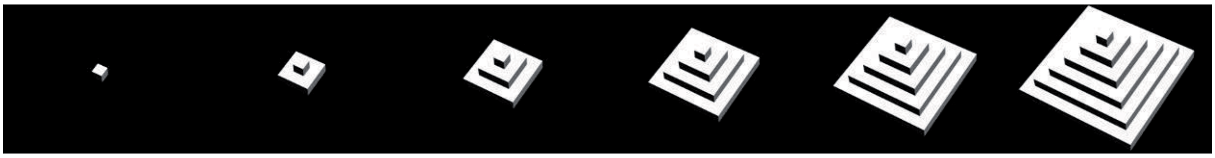

Fig. 10 A very simple 3D cellular automaton

When we work with 2D cellular automata (on orthogonal grids, with a neighbourhood of 8 neighbours, and rules concern the sum of neighbours (totalistic CA)) and when we display the results of the succeeding generations of the CA as layers, beginning with one cell, and going down, we also obtain pyramids. If we use only growing CA (no cell disappears), we can show the results by considering the « age » of each cell and translating that age either into a level of grey (from white for the oldest cells to black for the youngest) for each pixel in a bitmap (which is a kind of map) or into a height for each vertex of a mesh. Those are not true 3D forms, but only the surface of a 3D form. But it does not matter, as in a growing CA, no cell disappears, so there are no holes inside the 3D form obtained, and representing the surface of it is the same as trying to represent the whole form.
The CA that provides an actual, straight, pyramid is the most simple : no cell disappears, a cell appears if it has any number of neighbours (1 to 8)[1]. This CA shows growing squares, but with levels of grey, the picture is roughly the same as the previous distance map :

Fig. 11 Code 131071 cellular automaton (age of cell = level of grey)

With a mesh, we obtain a true pyramid :

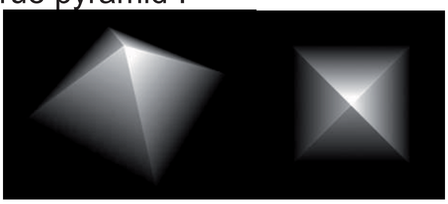

Fig. 12 Code 131071 cellular automaton (age of cell = height of mesh vertex)

Other rules yield more complex pyramids :

1 The decimal code of such cellular automata is obtained by writing the rules of the CA to form a binary number, then translating this number into a decimal number.

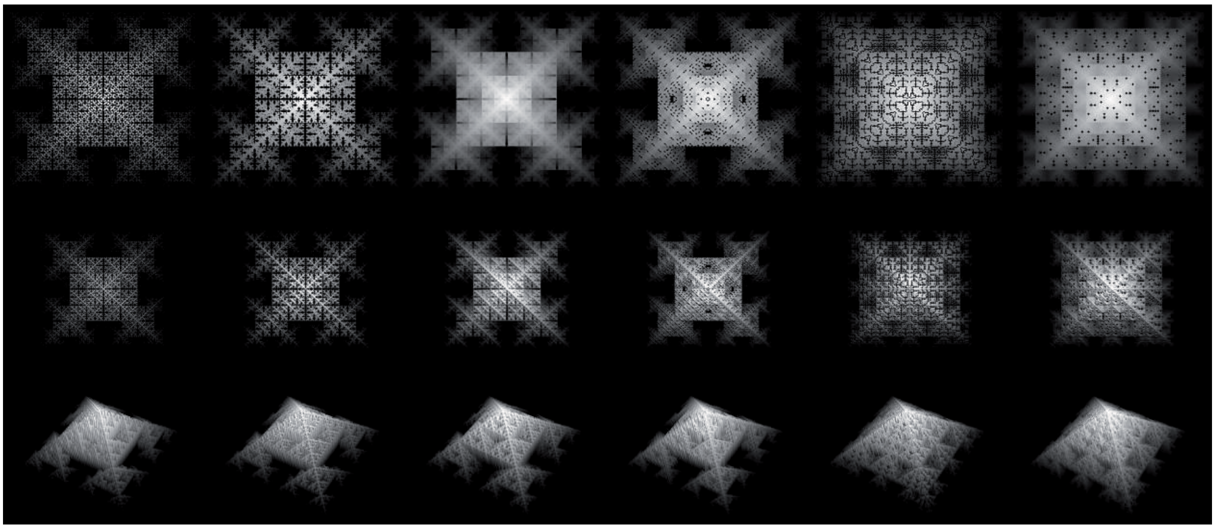

Fig. 13 Code 66047, 68095, 73727, 75775, 83967, 86015 cellular automata

Even with CA rules that are more complex than the one that yields the true pyramid, results show the same symmetries, as there is, first, the symmetry of the square in a 2D CA, and then the piling of generations and their regular growing that leads to a pyramidal configuration.

Fractals

Fractal geometry does not only let us generate forms that are not possible to make otherwise, but it makes us see forms in a different way. A cube is not only, as in traditional geometry, a figure with a particular set of symmetries but is the result, the *attractor*, of a particular set of contracting transformations (iteratively or recursively applied), and is (which is even more important) the only one. This set of transformation then characterizes better the cube than its set of symmetries. This way of seeing forms lets us recognize forms that are such attractors (they are self-similar) and forms that are not. In two dimensions, if we consider regular polygons, we can consider the equilateral triangle and the square as such attractors, but other regular polygons are not attractors of IFS. That has to do, obviously, with their capacity to tile that particular space. In 3D, only the cube (among regular polyhedra) both tiles the space and is the limit of an IFS.

This way of seeing forms induces us to look into forms to see how they are made with similar figures. When we deal with forms that are not self-similar, this generates incomplete, holed (and fractal) forms.

Looking into the pyramid (the one which is the half of an octahedron) we discovered that it can be partly filled with similar (half-size) pyramids but that there remained four holes : those holes are tetrahedra. There is a fractal form that is the result of an IFS constituted of 6 transformations, and that we can approach by iteratively (or recursively) applying this set :

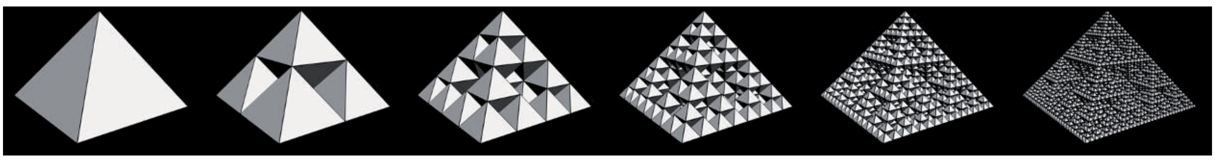

Fig. 14 Fractal based upon the filling of the pyramid with half-size pyramids

This construction is actually the half of the one based upon the octahedron.
We can also consider another IFS (pyr-1) constituted by 5 transformations which scale the form by ½, and translate the forms towards the corners of the whole form :

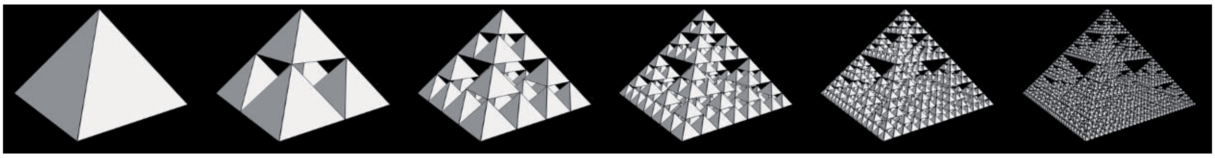

Fig. 15 Fractal based upon pyr-1 IFS

In a way similar to the one exposed in a previous paper [5], we can hybridize this IFS with another (pyr-2), defined by 5 transformations, with the same scaling, and which translates 4 forms towards the sides of the square base, and the 5th upon them, with a 45° rotation :

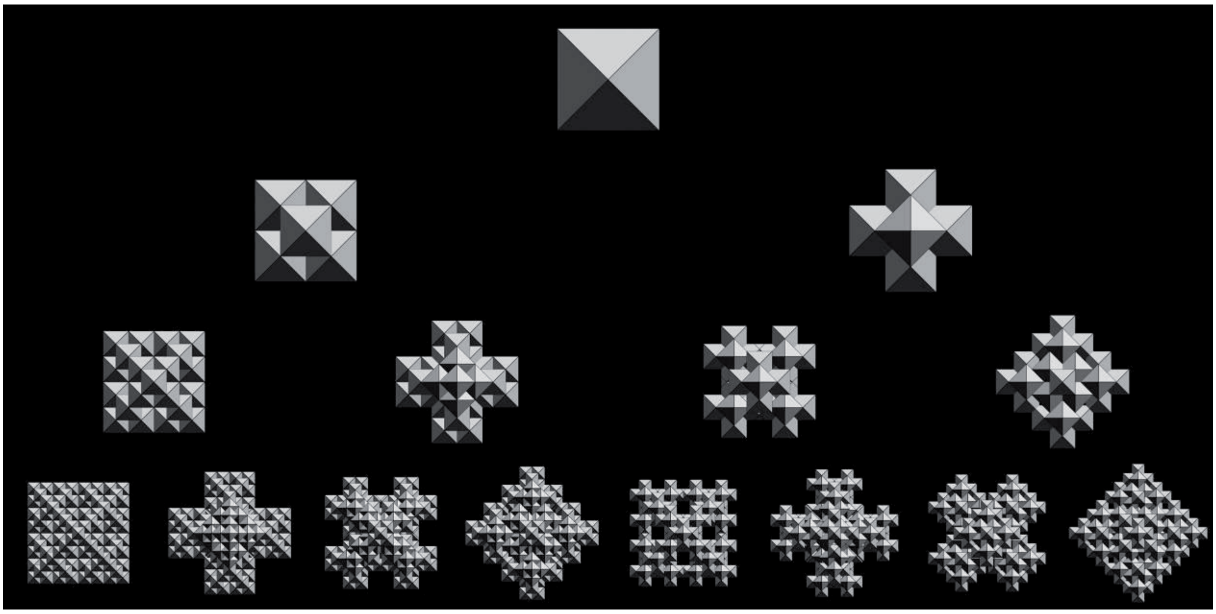

Fig. 16 Hybridization of pyr-1 and pyr-2 IFS (seen from above)

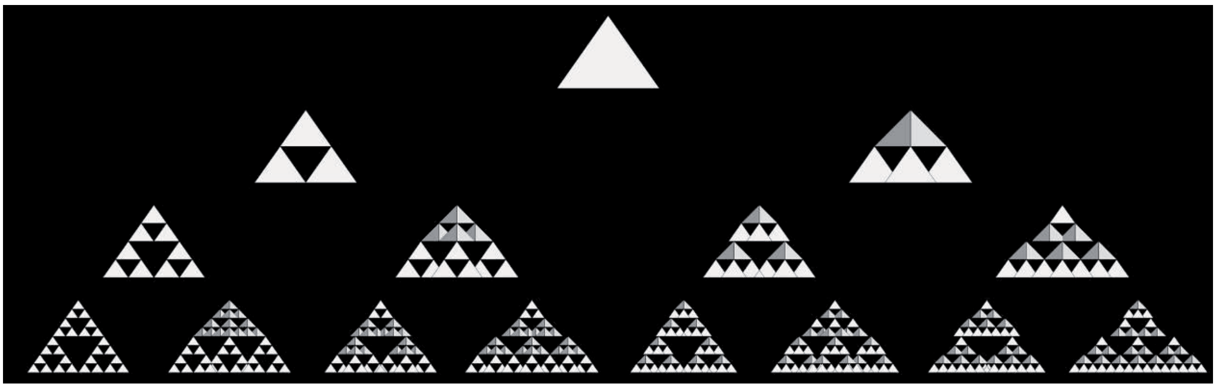

Fig. 17 Hybridization of pyr-1 and pyr-2 IFS (seen from forward)

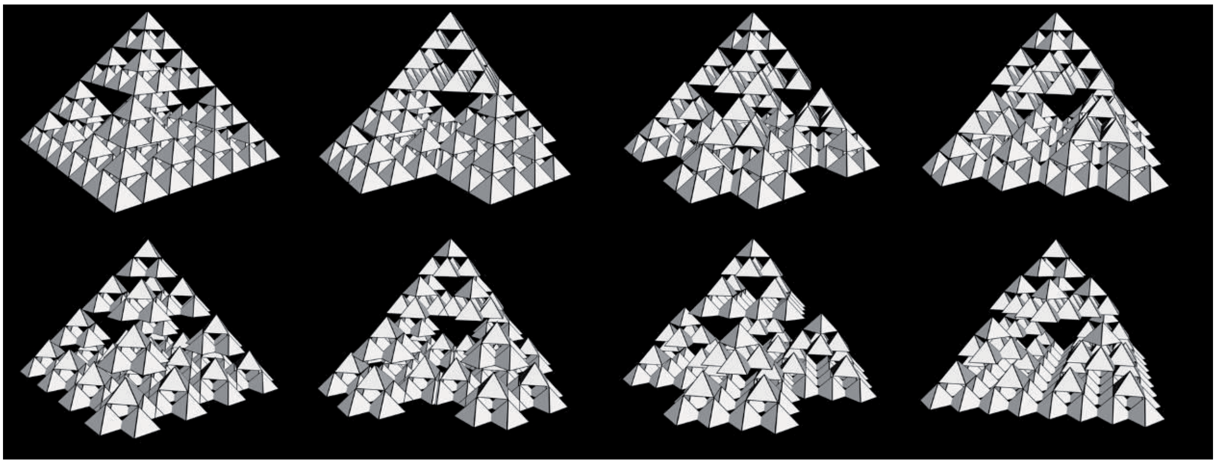

Fig. 18 Hybridization of pyr-1 and pyr-2 IFS (perspective)

A further step would be to find a way of playing with counter-forms as in 2D.

This little travel in the world of pyramids did not pretend to be exhaustive. Its only aim was to let us see pyramids in some different ways.

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