Using Gumowski-Mira Maps for Artistic Creation

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Abstract

The mathematical concept of fractal penetrates not only into numerous scientific fields, but also inspires the artistic creation, in particular the plastic arts. The report of direct or indirect derivation between the number of contemporary creations and the representation of the virtual objects, that are fractals, requires an attentive consideration in order to clarify the issues of the different possible transpositions of the concept, outside its ground area and the aesthetic meaning which it acquires. In this context, we propose some specific views of fractals in the artistic field. We suggest to solve two forms of derivation of "Fractalism": the technical derivation and the aesthetic value. We take as example the study of an Iterated Function System that provides chaotic maps: the Gumowski-Mira model.

1. Introduction

Art and science are two complementary fields which are both in relation with reality, the first is intuitive, the second is analytical. Indeed, "to make art a mathematics", an exact science, is to remove the artistic creation from the incertain perimeter of the free imagination and to subject it to the laws that regent rational paradigms : organisation laws, accuracy, etc., that are all related to the fields where technology finds its origin" [1].

This slope towards science finds one of its illustrations where the screen becomes the drawing board of a new type of artist whose function is to conceive an abstract imagery based on geometric formulae and mathematic algorithms.

The contribution of computers in the comtemporary artistic execution is remarkable. Art is entered in a new phase of experimentation, reinforced by the implementation of complex models such as "fractals". This new approach led to a redefinition of art and the creation of syntax and new artistic languages where transpositions and transformations such as form / formless, regularity / chaos, finite / infinite, etc., take place, similar to that which one find in other fields of knowledge [2].

Our intention is to make the distinction between the scientific and technical bases of the concept of fractality and its derivations with artistic claim. It is advisable here to wonder about the possibility to define the bases of a "fractalist aesthetic theory" starting from the scientific concepts which it takes as a starting point, and this, starting from the study of a very interesting iterated function system based on the Gumoswski-Mira model.

2. Theory of fractal

The contribution of the fractal geometry in science and art during the Seventies was strong and original. The fractal theory made it possible to open new fields of research and to widen the fields of mathematics and art via new and unexpected forms. But what one hears by "Fractal" and what are their theoretical references and characteristics?

2.1 Definition

From the Latin "Fractus", this neologism implies some specific properties such as irregularity, asymmetry, or symetry in scale. In contemporary geometry, one names by "Fractal", a two or a three dimensions space configuration characterized by "a given degree of irregularity and apparent disorder, whatever the scale of examination, that ordinary Euclidean geometry cannot satisfactorily takes into account" [3]. Because of their extreme irregularity at any scale and their broken and discontinuous nature, fractals are very complex geometrical forms.
Obtained by infinite regular fragmentation of a given image or pattern, fractals are also known by their paradoxical property : whatever the scale to which one view them, it remains impossible to accurately define their contours. Thus, fractal in the complex plane has an infinite circumference whereas its surface remains finite. For certain fractal curves, one can observe the same pattern at different scale : it is what one names autosimilarity is scale [4].

2.2 A short history

In 1960, Benoit Mandelbrot, a French mathematician who has worked on iterated function systems, started to develop a more systematic study of new complex forms that he called "fractals". In spite of the importance of this reaserch, fractals would not have had such a considerable repercussion and populaity apart from the field of mathematics, if the graphic representation of these new objects was not been made possible by computer technology.

In 1979 and 1980 Benoît Mandelbrot and his collaborators Sigmund Handelman and Richard F.Voss developed the first algorithm allowing 2D graphics representation of fractals. These pure mathematical abstractions became the icons of modernity while at the same time their scientific applications were still unknown by the general public. Although no artistic intention has drived the development of such computer generated images, it was clear that fractals have incomparable and unusual aesthetic and emotional potentials. Thus, fractal images were initially very popular for their aesthetic properties.

2.3 Characteristic and field of validity

A fractal object has at least one of the following characteristics :

- it has similar details on arbitrarily small or large scales;
- it is too irregular to be efficiently described using standart geometry;
- it is selfsimilar, i.e. the whole is similar to one of its parts.

The fractals do not have to satisfy all the properties mentioned above to be used as models. They only have to carry out suitable approximations of what interests a given field (the book of Mandelbrot : the fractals objects give a large variety of examples). Fractals may be deterministic or stochastic [5]. They often appear in the study of chaotic systems and can be divided into three main categories:

✓ Iterated functions systems (IFS). IFS are iterative processes which have the property to converge towards a fixed point, independently of their initialization. This point is called the "attractor" of the IFS. IFS produce fractals of which structure is, in most cases, described by a set of affine functions allowing to calculate the transformations applied to each point. These transformations are translations, rotations and homothetys, such as the ones observed in the Sierpinski triangle or the fern of Barnsley [6].

✓ Fractals defined by a relation of recurrence in each point of the complex plane. For example, in the Mandelbrot equation, for each value of C (complex constant), one obtains a succession of complex numbers that we can calculates the module. When the succession of the modules converges, C is considered like pertaining to the searched set, called the Mandelbrot set [5].

✓ Fractals that are based on stochastic and nondeterministic processes such as fractal landscapes.

Fractal art, as the world in which we live, is in permanent metamorphosis. Nothing is stable: neither the image, neither the forms, neither the color, nor the world they evoke. Fractals exhibe complex and chaotic universes, precisely characterized by proliferation phenomena of overload, saturation, or excess. Among fractals only those which are based on IFS have the property of autosimililarity in scale, meaning that their complexity is invariant by scaling.

One have passed to a culture of flow where instability, abstraction, displacement, and fugacity are the dominant characteristics. Fragmentation, irregularity, bifurcation and graining points connect regularity and chaos, random and foreseeable, finite and infinite [7]. The main characteristic of such practise is to minimize or reject the aestetic aspect of the artitic creation [8].

3. Derivation techniques and aesthetic value

Fractals works demonstrate an extreme diversity of materials, methods as well as the particular intentions of the artists. They have profound roots both in the geometry of Mandelbrot and the scientific theories of complex dynamical systems:

- The non-Euclidean fractal geometry developed since 1960 by Benoit Mandelbrot was previously studied by mathematicians as famous as CANTOR, PEANO, JORDAN in the 19th Century, then VON KOCH, HAUSDORFF, and BESICOVITCH in the first quarter of the 20th Century.
- The theory of recursive polynomial functions, developed between 1900 and 1930 by POINCARÉ, JULIA and FATOU.

One can observe that the integration of fractals in the artistic field is not only based on the technical processes. Thus, fractal images have also an aesthetic value that is based on both the richness and dispersion of generated colours and their forms themselves that sometimes reflect the real world such as Gumowski-Mira maps.

The artistic quality of fractal images resides in their originality and their diversity. Thus, in many cases the numerous solutions obtained with the same function system is very surprising. Although limited by their dependence to the technique, art works based on fractals induce an aesthetic upheaval. Indeed, they invite to a "pluriscopic vision of the world". According to Jean-Claude Chirollet: "Compared to the classicism of the cohesive rational order and of the totalitarian symmetry, the fractalistic aesthetics appears like its antithesis. Indeed, on the one hand, it asserts the artistic abandonment of the ideal of symmetry which appears like a lure in the light of the physico-mathematic theories of non-linear and chaotic processes, for which randomisation and junctions of no predicable trajectories play a crucial role. In addition, it supports the fragments and the details because fractal laws are not cohesives. However, these are laws of self dispersion and self decomposition of the « ad infinitum » fragmentation and of the structural recombining at any scale" [9].

One should note a double derivation, technical and semantic, of the aesthetic value of the fractals. Firstly, a fractal construction suggests the possibility to systematize and to privilege "a geometry of hailed, of sifted, of dislocated, of twisted, of tangled up, of interlaced" [10].

But fractalisation also mean the dispersion of the object unity with the profit of a multiplicity of profiles and of a profusion of details. By leaving the field of the computer graphics, the deployment of the figure is given up in the virtual infinite and true three-dimensionality is found. Indeed, the aesthetic value of the fractals can appear independently of the technical processes and mathematical approaches.

The integration process of the aesthetic values conveyed by the fractals does not only concern computer graphics. Thus, it gradually detaches from the technical contruction process to privilege the semantic aspect. In this context, the work of Miguel Chevalier represents both a transition between computer graphics and visual arts and between the technique and aesthetic. His interactive art works immerse the participants in fractal dynamic experiences such as repeated waves that oscillate according to the participants displacements. These art works are based on some forms of fractal processes that allow more sophisticated and enigmatic creative applications intended to question the participant.

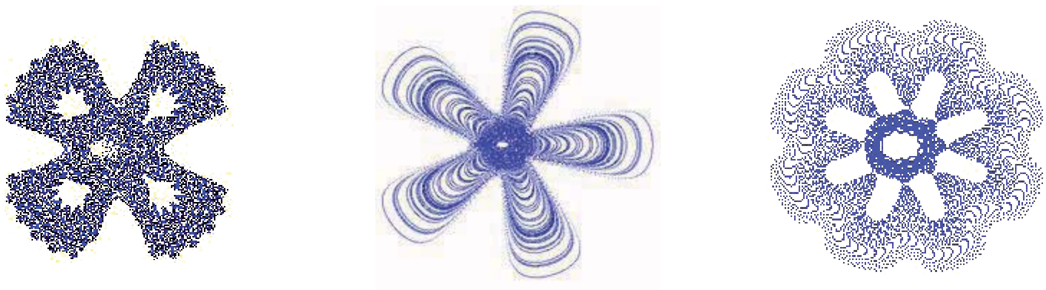
To conclude on this point, one can say that fractalist art is no more defined according to a construction technique nor by an integration of fractal forms within the art work than to the reference of aesthetic and significative values of fractals. One can even observe a double derivation of fractalism because of their intrinsic beauty and their capacity to mimic reality.

At this stage of derivation, there is no more a direct relation between art works and mathematical models, as it is acknowledged by Susan Condé : "There is not any artistic relationship between the art works presented here and the computer generated fractal images. The «fractalists» artists place their work in a metaphorisation and poetising scheme of fractality [11].

One can understand that the fractalist esthetism depends certainly on of the fractal geometry properties, but this derivation is nevertheless a metaphorisation process during which art gradually releases the meaning and the value of the process. The aesthetic value of the fractal geometry is not indissolubly attached to its mathematical consistency. The fractals have henceforth a new significance in each of their application domain and particularly in art. Undoubtedly, are the two processes of export of the concept and design not related to each other ? The systematic variation of scales leads to a fragmentist scientific philosophy which serve as a theoretical reference to the undefined aesthetics of fragmentation of the fractalist art objects [12].

4. The Gumoski-Mira model

Different nonlinear mathematical models which exhibit rich and complex properties exist in the literature. Some of them may reveal some aesthetic potentialities and are therefore interesting to study in the context of artistic creation. Among these models, one of the most interesting is the Gumowski-Mira model because of its very high sensitivity to the parameters (Eq.1) [9]. This model has been introduced for modeling and study accelerated particles trajectories at CERN in 1980. Iterations defined by Eq. 2 produce different kind of cellular patterns such as illustrated in Figure 1.



$A = -0.05, B = 1$ $\quad$ $A = 0.3, B = 1$ $\quad$ $A = 0.7, B = 0.99$

FIG.1 – Sample of Gumowski-Mira maps.

$$F(X) = AX + \frac{2(1 - A) X^2}{1 + X^2} \tag{1}$$

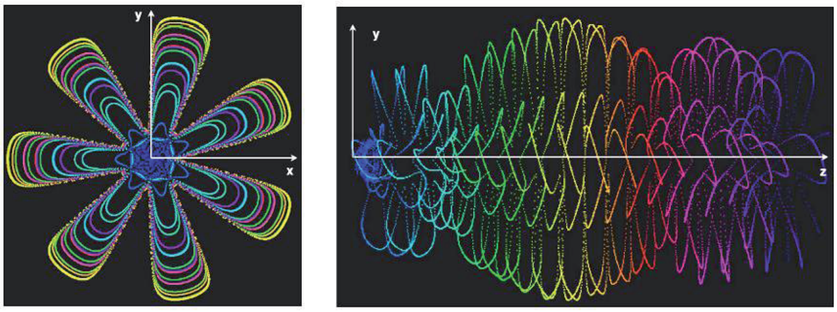
$$X_{n+1} = BY_n + F(X_n)$$

$$Y_{n+1} = -X_n + F(Y_n) \tag{2}$$

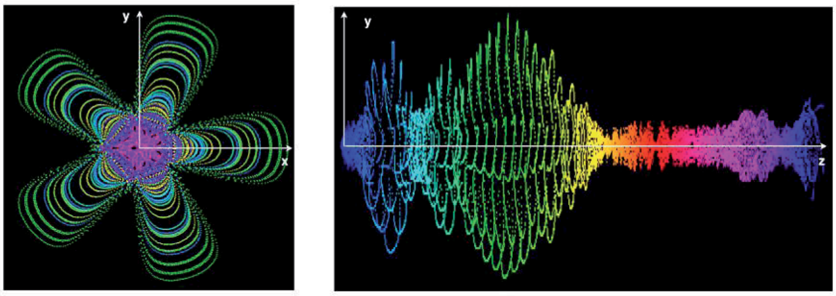
We observe that these patterns resemble very much (cross sections of) living marine creatures. Otsubo et al., performed a computer simulation on the Gumowski-Mira transformation and presented a variety of 2-dimensional patterns for different sets of the model parameters [12]. However, these patterns were monochromatic and did not take into account the colorimetric potential of the GM model.

4.1 Coloration of GM maps

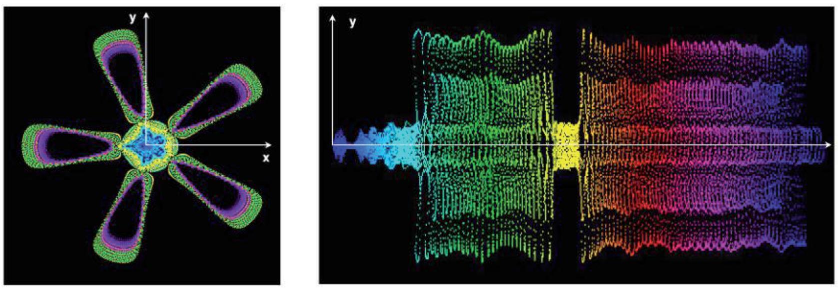
The role of color in artistic creation is crucial. We therefore started to study GM maps coloration. We observe on Figure 2 that the coloration of GM maps are not straightforward since the same color map gives different results (colors visible in the patterns). This is because GM patterns highly depend on the model's parameters. In addition, GM maps may exhibit either pseudo-periodic or chaotic behavior depending on the parameters value.



$x_0 = 0.5,\ y_0 = 0.5$; $A = -0.222,\ B = 1.000$



$x_0 = 0.5,\ y_0 = 0.5$; $A = 0.311,\ B = 1.000$



$x_0 = 0.5,\ y_0 = 0.5$; $A = 0.00,\ B = 0.305$

FIG.2 – Sample of colored Gumowski-Mira maps using the same colormap.

4.2 Morphing of colored GM maps

In order to develop some art works using the GM model, we have studied different rendering and viewing techniques such as 3D static and dynamic visualization. Among these techniques the dynamical morphing of GM maps is the more interesting one (Fig. 3). We start by selecting starting values for A and B, then we select two other values of the parameters. Finally, we run the simulation and observe a dynamic transformation of the GM corresponding to the starting values of A and B towards the GM corresponding to the end values of A and B.

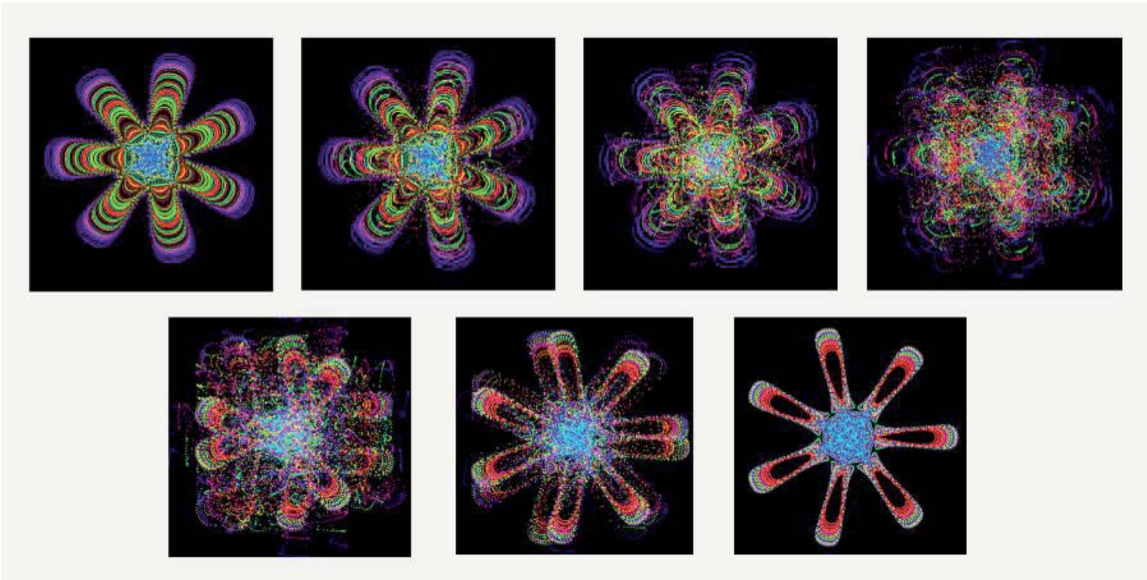


FIG.3 – Illustration of the proposed morphing technique for colored Gumowski-Mira maps.

5. Conclusion

The mathematical concept of fractal penetrates not only into numerous scientific fields, but also inspires the artistic creation. The report of direct or indirect derivation between the number of contemporary creations and the representation of the virtual objects, that are fractals, requires an attentive consideration in order to clarify the issues of the different possible transpositions of the concept. In this context, we propose some specific views of fractals in the artistic field. We suggest to solve two forms of derivation of "Fractalism": the technical derivation and the aesthetic value. We take as example the study of an Iterated Function System that provides chaotic maps: the Gumowski-Mira model. This model is very interesting because its very high sensitivity to the parameters study GM maps coloration. We observed that coloration of GM maps are not straightforward since the same color map gives different results (colors visible in the patterns). Indeed, GM maps exhibit either pseudo-periodic or chaotic behavior depending on the parameters value. We have proposed a new technique that allows dynamical morphing of GM maps. In the future we will propose different approaches that allow to better control the colour mapping in GM maps.

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