One can affirm that there are only 3D forms in the natural, material, world, bounded by surfaces without boundaries and impossible to remove from the forms, and consequently that lines (which would be boundaries of those surfaces) don't exist (not to mention points, which would be the boundaries of these non-existent lines).

Opposed to this extreme topological point of view, human beings, and architects particularly, through their means of representation, deal with shapes: they pretend to draw lines, to cut and fold surfaces (made of paper or other thin material); those pseudo-lines and pseudo-surfaces are not strictly speaking 1D, nor 2D, since they have some thickness. They may however be considered and conceived as such.

Forms constitute a space, as soon as we consider the physical and mental operations that we can make on them [1]; shapes constitute, as well as real forms, a space, i.e., a set on which we can imagine operations [2].

Contemporary architecture introduces the notion of diagram, which is obviously related to topological and geometrical operations on forms and shapes, as it appears in Peter Eisenman’s list: « extrusion, twisting, extension, interweaving, displacement, disassembling, shear, morphing, […] » [3]. Diagrams may be related to transformational grammars, and to generative processes [4].

This paper attempts to explore the signification of diagrams, and their relationship to operations on forms and/or shapes. It focuses on some diagrams, and discusses their use in generative processes, illustrated through works by the author.
Forms, Shapes, Diagrams, and generative Processes

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Abstract
One can affirm that there are only 3D forms in the natural, material, world, bounded by surfaces without boundaries and impossible to remove from the forms, and consequently that lines (which would be boundaries of those surfaces) don’t exist (not to mention points, which would be the boundaries of these non-existent lines).
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Forms constitute a space, as soon as we consider the physical and mental operations that we can make on them; shapes constitute, as well as real forms, a space, i.e., a set on which we can imagine operations.
Contemporary architecture introduces the notion of diagram, which is obviously related to topological and geometrical operations on forms and shapes, as it appears in Peter Eisenman’s list: « extrusion, twisting, extension, interweaving, displacement, disassembling, shear, morphing, […] ». Diagrams may be related to transformational grammars, and to generative processes.
This paper attempts to explore the signification of diagrams, and their relationship to operations on forms and/or shapes. It focuses on some diagrams, and discusses their use in generative processes, illustrated through works by the author.

Introduction
This paper has been trigged by an interest in Peter Eisenman’s EI-forms, which have fascinated me since I was confronted to his early projects as a student around 1980. Many years later, I want to question what has become an archetype of architectural form and confront it to my reflection on forms, shapes and generative processes.

1. Forms, shapes and operations

1.1 Distinction between forms and shapes
There is some difficulty for French or other Latin language people to distinguish between form and shape, which are both translated as forme or forma. Shape is of Old English etymology, and hasn’t penetrated our vocabulary, while form, of Latin origin, has been adopted by English speaking people. According to the dictionnary, form means forme in an abstract sense, and shape means it in a concrete one. But when we look at the reality of the use of these two words, we see that it’s not so obvious: shape is sometimes used in an abstract sense, and many occurrences of form are not abstract at all, and get back to the polysemy of the Latin word forma.
Letting aside the abstract and figurative meanings of these two words, one can
propose a radical distinction between the kinds of concreteness they imply: forms are what our kinaesthetic perceptive world is made of, and shapes appear through our visual perception, and through drawing. One can affirm then that there are only 3D forms in the kinaesthetic, material, world, bounded by surfaces without boundaries and impossible to remove from those forms, and consequently that lines (which would be boundaries of those surfaces) don’t exist (not to mention points, which would be the boundaries of these non-existent lines). Matter can be manipulated, forms can be made and transformed by operations like cutting, assembling, moulding, etc.

At the opposite from this extreme topological point of view, human beings, and architects particularly, through their means of representation, deal with shapes: they pretend to draw lines, to cut and fold surfaces (made of paper or other thin material); those pseudo-lines and pseudo-surfaces are not strictly speaking 1D, nor 2D, since they have some thickness. They may however be considered and conceived as such.

Forms constitute a space, as soon as we consider the physical and mental operations that we can make on them; shapes constitute, as well as real forms, a space, i.e., a set on which we can as well imagine and do operations.

For George Stiny [1], shapes are essentially 2D drawings, and what we see when we draw more shapes, or erase some of them, though those new shapes were not explicit in the first ones we had drawn. For example, if we draw a square (the outlining, the contour, of a square) and then another square which intersects it, we can see both squares, but also a new, smaller square (if the second square was translated along the diagonal), and two shapes that, referring to El-forms that will be discussed later, we can call El-shapes. Those new shapes do not belong as such to either square we have drawn, but they are however there to be seen. If we want, we can erase some lines to get only the smaller square (but no erasing will get only the El-shapes), or we can move the El-shapes one from another to give them their autonomy (but the smaller square will remain as a trace, as well as the larger ones).

The difficulty for Stiny is that the potentiality of El-shape is not present in the square (as outline). Indeed, the intersection is an operation that functions on the «true» squares, i.e., the surfaces, and not on the contours. But what we see when a line is closed is not only the line itself but the surface it bounds. The intersection has no actual meaning regarding the lines, but is easily dealt with if we consider the surfaces.

### 1.2 Cubes as forms

We can transpose the reflection that was made upon the pyramid [2] to the cube: one can make a cube by cutting stone, sawing wood, modelling clay, and so on. Those material forms may be displaced, i.e., translated or rotated; one can simply imagine scaling, but it is possible to do some other cubical form which is smaller or larger than the first one. Depending on the matter which has been used, transformations may be applied to that form: in clay, the cube may easily become a
sphere, or any other form; we can substract or add matter to that form, and so on. By cutting, we can even get two or more forms with the same amount of matter. From a topological point of view, what is at stake is only whether we get one or many forms, and whether this form is holed or not. We can make a hollow cube (by «removing» a cube inside a larger cube), or we can hole the cube from face to face to get some cubical torus. In the first case, no one will be aware of the reality of the form, at least by touching (though we may guess at its hollowness through its weight), because of our intrinsic tridimensionality; the second case is a very important topological event.

In any case, the surface of the cube or of either of its transformations is what stops our displacements, we can never go «inside» the cube, and if we cut it into two parts, the same happens with either one. This surface has no autonomy: we can «peel» a hypothetical cubical apple, but this peeling is not a surface as such, only a very thin 3D-form, and the peeled apple has got a surface as well as the unpeeled one. The surface of the cube, as that of any 3D form, is without boundary. We can travel on the surface of the cube, as well as on that of a sphere (like our terrestrial globe), without ever encountering the limit. Anyway, by touching as well as by seeing, we are actually aware that a cube is not a sphere: at some places, we touch or see a brutal change in the surface, which is one of the edges of the cube; we can follow this newly found entity, and touch or see a change in it, at one of the vertices of the cube. So, even if there are no lines as boundaries of the surface of the whole cube, there are actually lines, as boundaries of the surfaces constituted by the faces of the cube, and even those are divided into four lines which are the edges of the cube, which are themselves bounded by points, which are the vertices of the cube. This places the cube in the realm of polyhedra.

Though topology doesn’t consider measure as geometry does, it deals with those polyhedra and their number of faces, edges, and vertices. It is a topological rule that says that any simply connected polyhedron like a cube must have a number of vertices $V$, of edges $E$ and of faces $F$ that satisfy Euler characteristics: $V - E + F = 2$. In the case of the cube: $V=8$; $E=12$; $F=6$; $V - E + F = 8 - 12 + 6 = 2$. We must add that this relation does not imply that we deal with a cube as such, but with any hexahedron, i.e. a polyhedron with six faces, how much deformed it can be. But if we make a hole, for instance a square-section hole from a face towards the opposed face, we add 8 vertices, 12 edges, but only 4 faces to the first cube, and then $V - E + F = 0$, which characterizes this form as toroidal.

Even if the peeling of an apple, or a sheet of paper, are no actual surfaces, we are used to consider them as such, and we can make a cube by folding a conveniently cut sheet of paper. There are a few patterns that allow us to do that. Once folded and pasted, the paper encloses a part of space that we cannot penetrate, and is equivalent for our seeing and touching to the volume which is actually the cube. But we can also imagine being inside this folded paper, or, if the piece of paper is large enough, we can actually stay and be wrapped inside. We are then confronted to a cubical void, as we are, actually, very usually, inside a cubical room, once the door is closed.

All these considerations have to do with forms, as we encounter them in the material world, though when we introduced faces, edges and vertices, we began to abstract the material cube, and to pass from form to shape.

2.3 Cubes as shapes

Folding paper was already an abstract operation, because we cannot deal with actual surfaces (necessarily attached to volumes), but drawing is a next step.
Drawing a cube, we draw only the edges that we can see (never more than nine of them), and what we actually draw are some quadrilaterals (never more than three of them) which, by perspective, look like a cube. This supposes a lot of assumptions, and it is possible to draw something that looks like a cube from a unique point of view, and which is not one (it's even possible to fabricate such an anamorphic cube, and to photography this no-cube, which looks exactly like a cube). The cube is a very strong shape for our perception.

The cube as shape becomes an abstract entity, defined by characteristics of symmetry, and we can go on by naming cubical something that shares those characteristics, or some arrangement in which the cube is only a trace.

Computers have added to our means to represent forms and the numerical world is a space in itself. In this space, points and lines have a materiality as in drawings, but surfaces, which can only be approximated in the physical world for instance by sheets of paper, can actually be seen and manipulated, without loosing their characteristics of being without thickness.

Considering the cube now as something we can manipulate not only with matter, but by drawing, manual or computer drawing, we can make operations on it, such as: doubling (or cloning), translating, rotating, scaling, etc.; and as soon as we get at least two cubes: intersecting, removing, etc.

So we can easily transpose the experiment made on two squares and imagine, and even actually make, two cubes intersect and see what we get. We obtain the two first cubes, that remain, but also a smaller cube (supposing the second cube has been translated along the great diagonal of the first), and two new forms, or shapes, that will be called El-forms.

2. Diagrams

2.1 What are diagrams?

A diagram is, etymologically, «a geometric figure, that which is marked out by lines». For Peter Eisenman, who is the leading architect for the introduction of diagrams in contemporary theory of architecture, «while it can be argued that the diagram is as old as architecture itself, many see its initial emergence in Rudolf Wittkower’s use of the nine-square grid in the late 1940s to describe the Palladian villas.» [3, p. 27]. Eisenman then asks «what the difference is between a diagram and a geometric scheme. In other words, when do nine squares become a diagram and thus more than mere geometry?» [3, p. 27].

The difference is that a diagram is not merely descriptive or analytical, but generative. Even if Eisenman doesn’t use this term in exactly the same sense as it is implied in generative art, this generativity of the diagram is what interests us

Eisenman insists upon the concrete, palpable nature of architecture, as opposed to the abstract, conceptual process of design: «As a generative device in a process of design, the diagram is also a form of representation. But unlike traditional forms of
representation, the diagram as a generator is a mediation between a palpable object, a real building, and what can be called architecture’s interiority.» [3, p. 27]
Especially, «[the diagram] suggest[s] an alternative relationship between the subject/author and the work. Such an alternative suggest[s] a movement away from classical composition and personal expressionism toward a more autonomous process.» [3, p. 169] This concern meets up with issues regarding generative art [4].

*Diagram diaries* [3] classifies diagrams through two means: first, a distinction is made between «diagrams of interiority» and «diagrams of exteriority»; to simplify, the first ones rely on the architectural form(s) as such, while the second ones rely on external data. Diagrams of interiority are divided into four categories: grids, cubes, El-forms and bars. Diagrams of exteriority consist of four types of data: site, texts, mathematics, science.

Secondly, a list of 39 tools is given, related to 40 projects: one project may rely on several tools, one tool may be used by several projects. Some tools are used in only one project, while some other tools may be used in as much as eleven projects.

The use of some tool in some project is indicated by a sign, which is differentiated along its as a diagram of interiority or exteriority. Some tools are only related to interiority or exteriority, respectively, while other ones may be related to both, sometimes even in the same project.

The 39 tools are divided into two categories: 24 «formal» tools and 15 «conceptual» tools. This distinction is autonomous from the distinction interiority/exteriority. There is no question about the first category, but one may wonder why «voiding» or «folding», for example, are classified as conceptual tools, as they seem to rely on formal operations.

Tools are not explicited, they are named, and exemplified by the projects. The names are interesting, because they refer clearly to *operations*, either by using the suffix -ion (extrusion, extension, intersection, and so on) or the suffix -ing (twisting, interweaving, disassembling, and so on), with six exceptions (displacement, shear, interference, slippage, montage, laminar flow).

One can wonder about the constitution of this list of tools. There is a tool named transformation, which is a very vague name and could cover a lot of operations, and other tools which could be kinds of transformation, as extrusion, extension, torquing, distortion (in itself not very explicit), warping, and so on. In the same way, there is a tool named displacement, but we find also rotation and slippage which could be considered as kinds of displacement. One can wonder about the difference between the formal tool «disassembling» and the conceptual tool «decomposition».

### 2.2 El-forms as diagrams


It must be noted that the tools mentioned earlier are not implied in the making of the El-forms themselves; they are used on El-forms, which are given without being explicated, or anyway not by using those operations.

Most El-forms are based on the cube (they are the only ones that will be considered here): those El-forms occur the most clearly in projects (1) to (7), and are of two kinds. The simplest one appears in (1), (4), (5) and (6):
Fig. 3: El-forms of the first kind as they appear in (1) and (6), (4), (5), (7)

The most complicated kind of El-form appears only in House 11a, which was a project for Kurt Forster, and in Cannaregio Town Square, where «scaled versions of the house were placed in the matrix of voids» [3, p. 177].

Fig. 4: El-form of the second kind as it appears in (2) and (3)

2.3 El-forms as results of operations

The first kind of El-form can be described as the result of the intersection of two cubes considered as shapes as in Fig. 2, but also as half a cube, i.e. the three faces (supposed to be thickened, or extruded) linked to a vertex. In the latter description, one can remark that the El-form is what is only really drawn when a cube is represented in an axonometric projection (or in a perspective, but Eisenman generally favours axonometric projections). Eisenman here clearly plays with shapes, or with drawn representations. This is confirmed by the use of projection itself in House El Even Odd, which is a kind of 3D anamorphose.

The first kind of El-form may also simply be described as a cube from which one smaller cube sharing one vertex with the first one is substracted. The status of the form and of the void change with the size of the cube we remove relatively to that of the first cube. The trace of the two cubes remains, because our perception tends to complete the larger cube, and to complete the void as a smaller cube, this time not a cubical volume, but a cubical void:

Fig. 5: the first kind of El-form

This first kind of El-form may be extended into the second kind kind, by removing another cube issued from the vertex diagonally opposed to the first one:
Subtraction is the operation involved, which is a material operation (one can do it with clay, or even some harder matter), or, more abstractly, a boolean operation (along with union and intersection). If the subtracted cubes are small enough, it does not change the topological status (order) of the polyhedron. \( V=14; E=18; F=12; V-E+F=14-18+12=2 \). But, if the smaller cubes are large enough, the cube becomes the topological equivalent of a torus, which is a very strong transformation, as the Euler characteristics changes abruptly: \( V=32; E=20; F=12; V-E+F=32-20+12=0 \).

We can then consider that there are actually three species of El-forms: a cube minus one cube, a cube minus two cubes, and the toroidal El-form. The passage from the second to the third species is what can be called, in René Thom’s terms, a catastrophe.

This latter occurrence of toroidal El-form may also be considered as a folded square-section tube, or, in other words, as a folded line along which a square generates a form:

This interpretation is relevant, as in another project, *Max Reinhardt Haus*, Eisenman uses this kind of operation.

Returning to the toroidal El-form as the result of two boolean subtractions (one larger cube minus two smaller cubes), we can extrapolate and, considering that *minus* by *minus* gives *plus*, subtract from the larger cube \( A \) only the parts of smaller cubes \( B \) and \( C \) that don’t belong to the intersection of \( A \) and \( B \). Or, in boolean terms (or rather in set theory), instead of \( A \setminus (B \cup C) \), consider \( (A\setminus (A\cup B))\cup (A\cap B) \). The result is that an even smaller cube is *nested* inside the El-form, and that the voids created are themselves El-forms of the first species.

We can also consider another way to get El-forms. Gridding is one the conceptual tools listed by Eisenman and grids are a category of the diagrams of interiority. One
can grid a square (and the nine-squares grid was at the origin of the idea of diagram) but one can grid a cube too, as 8-cubes, 27-cubes, or any $n^3$-cubes grid. In *House IV*, Eisenman transforms an 8-cubes grid into a 27-cubes grid, and explores a lot of possibilities based upon this decomposition. One of them consists in removing seven smaller cubes in the same way as in the generator of the Menger sponge. With that point of view, El-forms may be seen as the gridding of a cube, in which we remove one smaller cube situated at a corner (or two cubes situated at diagonally opposed corners). It may seem that it is not so different from the subtraction of a cube without gridding it first, but it is different because gridding enhances the fact that the cube is self-similar, is composed of $n^3$ cubes n times smaller.

![Fig. 9: 8-cubes grid and 27-cubes grid leading to El-forms](image)

To be complete, we may add one more description of the cube and El-form. A cube of edge $a$ (with $b+c=a$) is composed of two cubes of side $b$ and $c$ respectively, three $b \times b \times c$ parallelepipeds, and three $b \times c \times c$ parallelepipeds. Removing one or two cube(s) from this decomposition is another way of getting some El-forms.

![Fig. 10: decomposition of the cube and El-forms](image)

### 3. Generative processes

#### 3.1 Coding El-forms

In this part we focus on El-forms of the second kind (and especially of the third species). Referring to the different ways we have described those El-forms, there are many systems we can choose to code them. Each case may lead to very different variants.

A cube from which two smaller cubes (equals or not) are subtracted at opposed corners leads to a first type of code. This can be realized as a kind of IFS, in which one starts from a cube, produces two cubes by the adequate transformations, and
then subtracts them from the first cube. The extrusion of a folded line can be coded in two ways. First, it can be considered as a line along which a square slides. Or, one can borrow L-systems with the following vocabulary and two examples of rewriting rule:

- **C**: make a cube
- **F**: go forward
- **B**: go backwards
- **U**: go up
- **D**: go down
- **L**: go left
- **R**: go right

[C → CFCFCUCUCLCLBCBCDCDCR](1)
[C → CFCFCLCLCUCBCCBCRCRDCD](2)

This code supposes a gridding of the cube. The gridding of the cube, which is related to its self-similarity, leads also to the use of IFS, which are based upon this characteristics. We have to find the transformations which replace a cube by the \(n^3\) smaller cubes that compose it, excluding those which correspond to cubes at opposite corners. Those transformations are very simple: they are all homotheties (scaling) with a rate of \(1/n\), composed with translations putting the cubes on the right places.

In the same way, we can find the transformations that replace a cube by 2 cubes and 6 parallelepipeds that compose it, and then exclude the cubes. Here the scalings are affinities (which means that the rates may be different for x, y and z).

### 3.2 Scaling and nesting

Scaling and nesting are part of Eisenman's tools. The nesting may be considered in the voided cube that appear at the core of the toroidal El-form:

![Fig. 11: nesting of El-forms (first variant)](image)

In this case, there as many El-forms as steps of the process. Or we can consider the two cubes that have been removed from the first, and nest a new El-form in each of them:

![Fig. 12: nesting of El-forms (second variant)](image)

In this second variant, the number of El-forms is \(1+2^n\) for \(n\) steps.

### 3.3 Recursivity
Even if scaling and nesting do not lead for Eisenman to recursivity, it is very tantalizing to use El-form coding recursively and see what we get...

L-systems are rewriting systems and lead naturally to recursive El-forms. Using the rewriting rules seen earlier, we can obtain some recursive El-forms:

![Fig. 13: results of 3 first steps of application of rules (1) and (2)](image)

We can hybridize [4] those two rules to get much more results:

![Fig. 14: results and cumulated results of steps 2 and 3 of hybridization of rules (1) and (2)](image)

**Conclusion**

This exploration was not aimed at reproducing Eisenman’s architecture, but intended only to try to interpret El-forms as results of operations, and better understand the operations we can do on forms.

One could exploit those operations on diverse forms, as Paul Coates for instance does by interpreting Le Corbusier’s Domino house with translations and subtractions [5]. This can lead to genetic algorithms, an issue we have not explored here.

**References**