

Generating Architectural Envelope through Mathematica

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Abstract

After millennium, the making of building form raises and concerns sustainable terms. The architecture with an emphasis on sustainability is often devoted to a greater division of exterior and interior space by adding artificial devices on building envelope; such as the solar panel, wind turbine, and new architectural envelope material (GPR). Can this kind of building devices designed for environmental modulation satisfy the needs of sustainability? D'Arcy Wentworth Thompson ever comments in ***On Growth and Form*** that the morphology of living form has a dynamic aspect, under which the living forms are able to construct and dynamically maintain themselves to adapt the environmental change [1].

In the past, mathematics is often employed by engineers and architects to simulate and translate the form of 'nature' into geometric shapes. Now, architects are confronted by a more complicated challenge from 'nature': climatic change. There is an increasing interest in the new way of generating architectural envelope or surface. It encourages and inspires research on the investigation of the morphogenetic process between forms and dynamic forces through an algorithmic program – 'Mathematica.' It also suggests that the architectural design incorporating the issues of 'nature' (e.g. Airflow, Light, and Heat) in the generative manner may result in a more responsive form to nature. In order to examine the feasibility of the above propositions, the proposed paper takes airflow as subject to explore how 'Mathematica' can generate architectural envelope via environmental issue. The paper draws parameters from airflow to frame a time sector of vector field, which constitutes a basis for the production of a generative line. Following that, the time sectors are laminated together, and the generative lines are used to compose the envelope to represent the consequence of environmental variations.

This paper doesn't aim at proving how accurately the algorithmic approach to the challenges from environmental issues. Instead, it attempts to argue and evoke that the value of envelope lies in the interplay between environmental challenges and architectural design process. Through the interplay, an architectural form would embody functions or activities associated with envelope to create a new relation with 'nature'.

1. A mound of termite

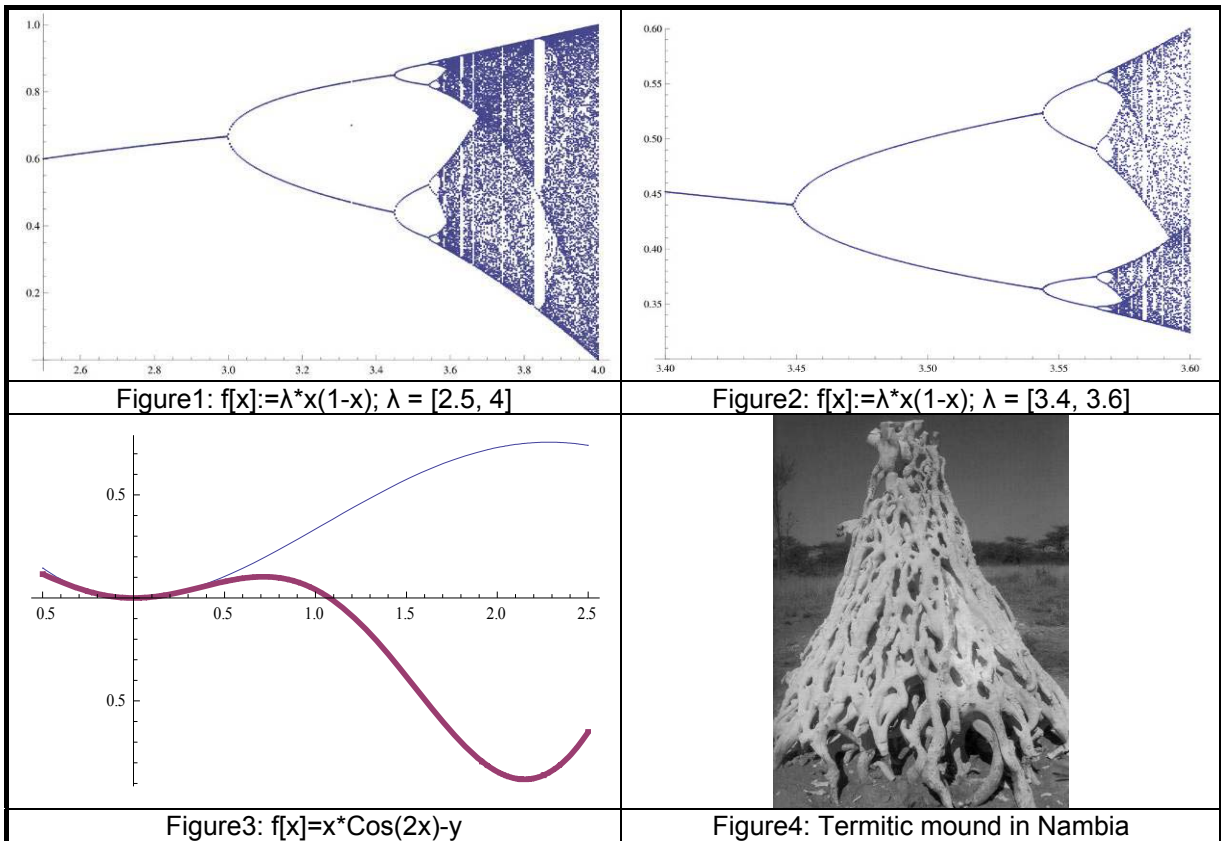
A mound of *Macrotermes michaelseni* termite in Nambia has shown some potential to establish the relations between envelope and comfortable living form through the dynamic force of environment. While the termite mound have been investigated for over a century, the form of nest is proven to relate to the function of ventilation generated by the pressure fluctuation resulting from the changes of wind direction and speed in the dynamic environment outside [2]. Moreover, even though many studies indicate that there are absolutely interactive relations between form and environment, the morphogenesis still remains as a field for exploration.

The mound is organized by the bifurcate conduits to integrate the external wind pressure and internal buoyancy. This formal prototype is helpful to insulate the dry air from outside and maintain the temperature inside. Certainly, the bifurcate conduit becomes the most important functional pattern to determine the morphology of termitic living form. Does this mean that the termite mound transfers its

architectural form to functional envelope via operating the energy in environment? In order to study the relations between environmental energy and morphology, research tries to use logistic map popularized by the biologist Robert May and written down as an equation by the mathematician Pierre François Verhulst to simulate and interpret the meaning of form of termite mound. Originally, the form of termite mound is determined by bifurcations occurred in different energy gradations to disperse overly concentrated energy and keep system stable. Michael Weinstock also argued in *Matabolism and Morphology* that fluid energy transportation in particular is an essential determinant of body plan and overall morphology [3]. In order to depict the relations between energy and morphology of termite mound; Figure1 is rendered by mathematica through logistic map given by equation (1) to show how the form forked and generated.

$$x_{n+1} = \lambda x_n(1-x_n) \tag{1}$$

In equation (1), n denotes a generation, x_n denotes environmental variation in generation “n”, and λ denotes an environmental parameter of energy in generation “n”. By the calculation of logistic map, two different bifurcation diagrams are derived shown as Figure[1&2]. The first diagram calculates the parameter λ in region [2.5, 4], the second one does the parameter λ in region [3.4, 3.6]. Comparing with these two diagrams, it is noted that different regional energy makes the first bifurcation occurred at different energy gradation ($\lambda= 3$ and $\lambda= 3.45$) and influences the whole morphological structure. All of these forms could be as a model of adaptation to the energy. Through the adaptation of energy, an optimal morphology is generated to respond to the environmental variations, and it can be shown as a set of generative lines. The morphology of logistic map is similar to the structure of termite mound’s envelope. The bifurcations divide the mound into several energy regions in height and orientation to derive different air density, which is the main factor to drive inner convection [4].



In order to figure out the restrictions of equation (1) in explaining the relation between environmental energy and morphology, the study attempt to modify and rewrite it as follow:

$$f[x]=x*\text{Cos}(2x)-y \tag{2}$$

Equation(2) is rendered by 「Picard Iteration Method (PIM)」 and shown in Figure3. Through the comparison if with Figure1 and Figure2, it is found that the bifurcation form could be rendered from different equations. It means that logistic map is one of the basic equations to represent the morphology of termite mound. Although it can not be used to explain the morphology in particular place or environment, it shows that the morphology of termite mound hold strong relation to the fluctuant energy in environment. According to the observation above, this study attempts to frame some key points below to establish a method for transferring natural environment to energy environment. It also associates with morphological study of architectural envelope in response to place. To sum up, the assumptions about this study are listed below:

1. The architecture could be regarded as a monolithic organization directed or manipulated by environmental energy.
2. The interactions between generative line and energy are a prototype to explore feasibility about a morphologic study of architectural envelope.
3. The environment could be represented and simulated by mathematics.

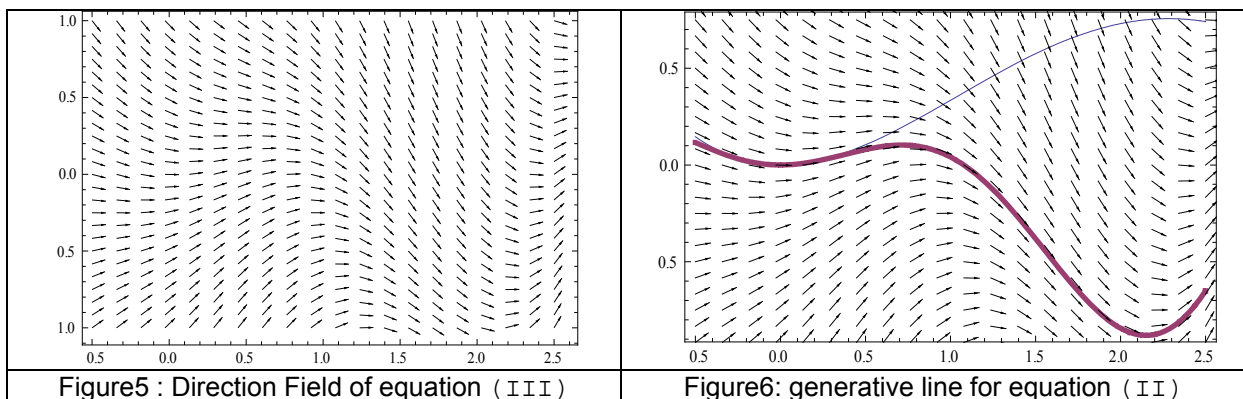
2. Generative Line

Through the morphological analysis of termite mound, the question is not only to indicate whether the environmental issues generate the form, but also to show the generative line could be a basis to help designer to contour the other field to study architectural morphology. In order to understand the process how environmental energy originate generative lines, the study tries to plot generative line and energy flow about equation (2) in two-dimensional transformation and give a process consists in calculation before piecing together a three-dimensional object out of them.

Firstly, the study tries to plot a map to visualize the flow of environmental energy about equation (2); consequently, equation (2) is formulated as a differential equation (3) to get vectors described by roots (x_0, y_0) .

$$dy/dx = x \cdot \cos(2x) - y \tag{3}$$

Conceptually, if the roots could be gotten from equation (2), then, the roots could be generated as a “Direction Field” (Figure5) to show dynamic streams of environmental energy. On Figure5, all roots designate a tendency shown as curves traced by the arrows on “Direction Field”. However, all of these curves are symbolized as approximate solutions. But, there is still an optimal curve existed in “Direction Field”, How to diagnose it?



Research uses 「Picard Iteration Method (PIM)」 to get a optimal curve about equation (2). By algorithm of PIM, equation (2) could be extended into several generations until the optimal morphology generated to fit in “Direction Field”. By the alternation of six generations, equation (2) could be solved and generated as differential equation (4).

$$dy/dx = 1/960 (-135+x (135+2 x (-15+x (5+(-5+x) x)))+15 (9+13 x) \cos[2 x]+15 (-11+26 x) \sin[2 x]) \quad (4)$$

When differential equation (3) and differential equation (4) are plotted by PIM, the blue curve and purple one is generated respectively for them in Figure6. Comparing with two curves, the purple curve is fitted in "Direction Field"; it means that curve is generated as an adaptable morphology after six generations. Learning from PIM, the algorithm could help study to generate an optimal morphology in energy environment, 「Equation」, 「Direction Field」, and 「PIM」 have become a investigative tool kit to associate a morphological study with energy environment. Before starting with this tool kit, the most important thing to do is try to define an equation based on the data of local environment.

3. Environmental Equation

This Paragraph is aimed at transferring natural environment to energy environment; in natural environment, wind speed, pressure and wind direction are the

Time	Pressure (hPa)	Wind Speed (m/s)	P(Avg.) -P	Wind Direction
06:00	989.0	5.1	-5.75	π/8
07:00	988.6	5.3	-5.35	π/8
08:00	988.1	4.5	-4.85	π/8
09:00	986.9	4.3	-3.65	π/8
10:00	985.7	4.5	-2.45	π/8
11:00	984.1	5.2	-0.85	π/8
12:00	981.0	5.8	2.24	π/8
13:00	980.1	8.4	3.14	π/8
14:00	978.9	6.1	4.34	π/8
15:00	976.9	7.9	6.34	2π
16:00	976.4	7.1	6.84	2π
Avg.	983.245	5.836		

Table1: 28/09/2008 P-V Data

critical factors to generate particle movement as a streamline in Bernoulli's Equation.

Bernoulli discovered that the dot product of pressure and wind speed is a "mechanical work", which is an environmental energy transferred by forces. It means that natural environment could be represented as an energy environment. The mechanical work could be formulated as a product by Pressure (P) and Wind Speed (S), which is given by:

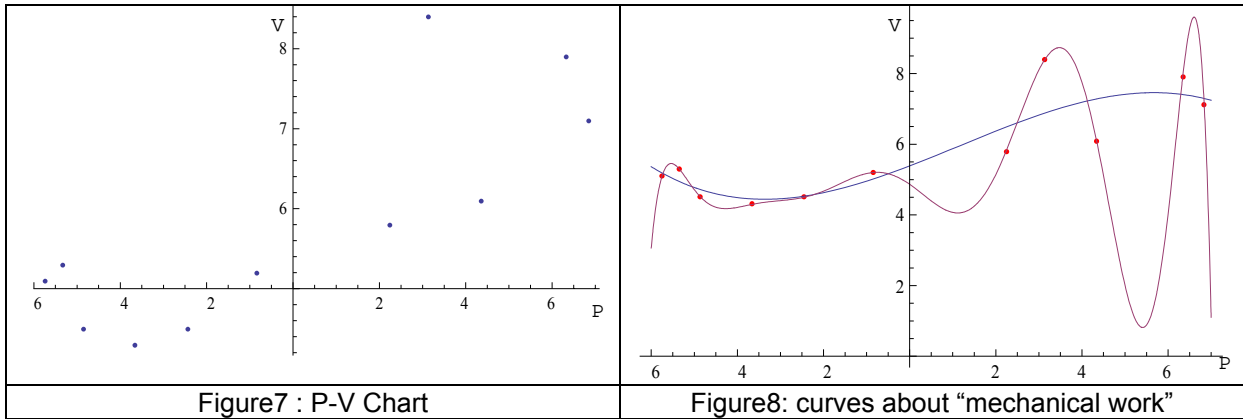
$$W=P*S*\cos \theta$$

From the inspiration of Bernoulli's Equation and mechanical work, the

environmental equation could be conducted by wind speed and pressure. But, how to formulate these two factors to define an equation? The study uses the data on Table1, which is derived from "Central Weather Bureau" and associated with wind speed and pressure in "Taichung Harbour" in Taiwan. In mathematica, the data of 「P(Avg.)-P」 to 「Wind Speed」 could be plotted as a chart (Figure7). Due to the rule of mechanical work, if there is an equation (x) existed to connect all dots in Figure7, 「P(Avg.)-P」 and 「Wind Speed」 would be explained as the roots of equation (x), then, equation (x) is defined as a environment equation for this energy environment to help study to plot a "Direction Field" and generate a optimal morphology. In mathematica, function 「Fit」 could help research to define an equation to approach the analysis of data. 「P(Avg.)-P」 and 「Wind Speed」 is calculated as a product, which is symbolized by parameter 「x」 in polynomials given by equation (6) and equation (7). Both equations are approximate solutions for Figure7, in order to diagnose which is the optimal solution for energy environment; both equations are plotted as curves on Figure8.

$$f[x]= 4.86632 -0.844849x-0.368324x^2 +0.314711x^3 +0.134932x - 0.0198913x^4 -0.0114207x^5 +0.000327375x^6 +0.000352525x^7 +5.95609 \times 10^{-6} x^8 -3.6747 \times 10^{-10} x^{10} \quad (6)$$

$$f[x]= 5.38681 +0.468156x+0.0283144x^2-0.00818202x^3 \quad (7)$$



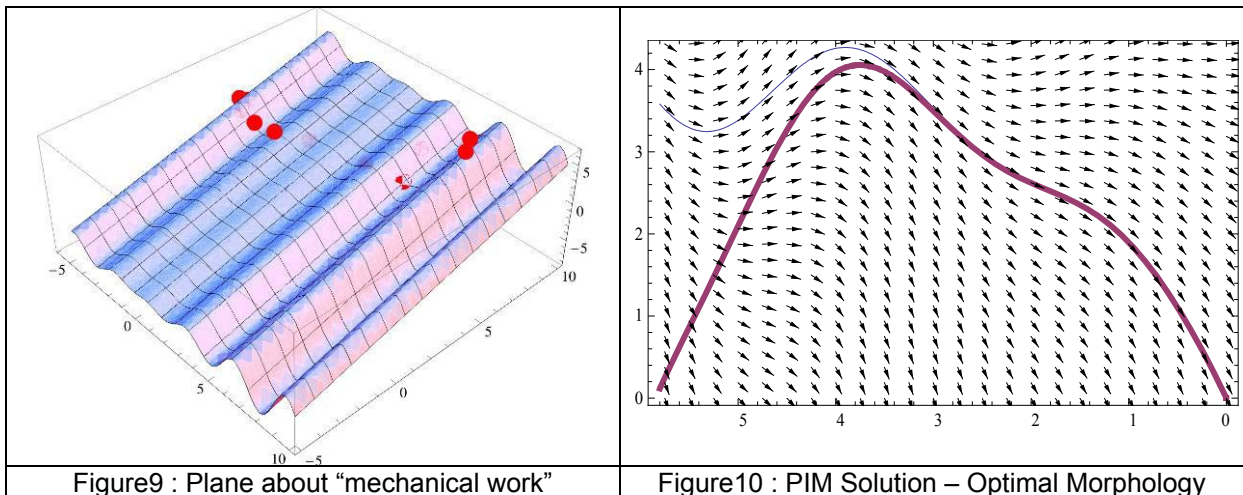
In Figure8, equation (6) is plotted as purple curve, and equation (7) is plotted as blue curve. Comparing with two curves, the purple one is penetrated through all dots. It means that equation (6) is the optimal solution used to define as an environmental equation. The curve on Figur8 is a reference for study to sift the optimal environmental equation from multiple feasibilities, it doesn't relate to the morphology. From the experiment in chapter "Generative Line", the morphology should be derived from environmental equation in differential.

4. Generative Envelope

Actually, Figure8 could be plotted into three-dimension (Figure9) to modify equation (6) to equation (8). In this paragraph, the parameters of pressure, wind speed and wind direction is denoted as a value of [x, y, z]. Equation (8) is given by:

$$f[x]=-2.56+0.64y+0.31x*\text{Cos}[2x] \tag{8}$$

In Figure9, the red dots show the relation of pressure, wind speed and wind direction, and the curved surface is a reference to help study to diagnose how closely that equation (8) approaches to exact solution about environment equation. From the exploration of last paragraph, equation (8) could present a solution of energy environment. At this step, equation (8) could be assisted with PIM to originate a generative process to visualize the inner structure between morphology and fluidity of energy environment. In Figure10, blue and purple curve presents an algorithm of morphology in different generation. Especially, blue curve approaches the optimal morphology. The curves are like the bifurcate prototype of termite mound's envelope or the mold of Gothic. By the alternation of generations, the morphology of termite mound is aggregated by numerous bifurcation prototypes. Consequently, Figure10



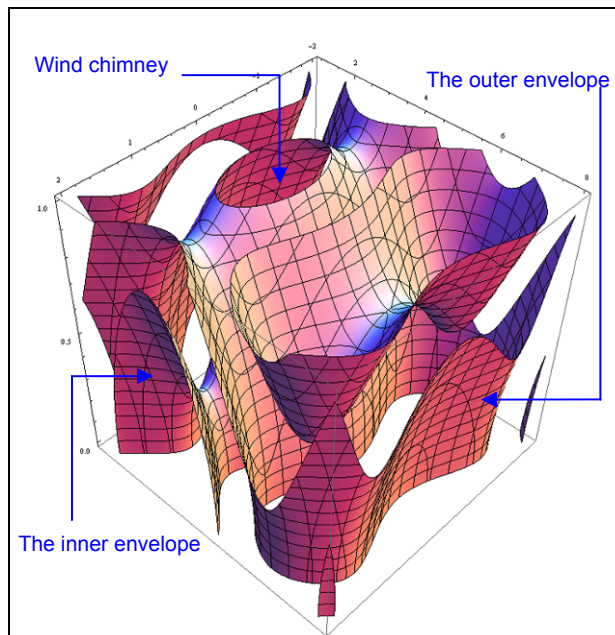


Figure12 :
the generative architectural envelope

doesn't show the result of whole, it just a partial mold of architectural envelope. In order to examine how the curves aggregated to generate an architectural envelope, the study derives more optimal curves as architectural molds from the data of "Central Weather Bureau" on August to October in 2008. The study takes a daily optimal curve as a mold of architectural envelope. By the aggregation of optimal curves, the architectural envelope is generated and shown on Figure12. The algorithm of envelope reveals the features of double skins and wind chimney. The basic knowledge of the experiment of wind tunnel explains when temperature of outer envelope is warmer than inner one; the different temperature on skins will engender the wind to flow through the interval between two envelopes; the wind chimney would accelerate an efficiency of ventilation. It seems that the generative architectural envelope originated in parametric algorithm is meaningful, but how to present the rationality of this meaning?

Moreover, even though the study does not aim at explaining how this morphological research really deals with a specific time and place, but, the study attempts to frame a field to explore the method how the morphology of architectural envelope could be generated and modulated by energy environment. By the aggregation of generations, the morphology of architectural envelope will continue to be generated.

5. Conclusion

After millennium, the most important global issues are focused on sustainability, seeking through the environmental issues, sustainability attempts to search a new architectural prototype to respond the questions in this green age. The ideas embedded in sustainability are often summarized under the terms generative, self-organized, responsive and circulative. These four issues form more complex behavior and depict that a number of simple prototypes could be operated and emerged from an environment. Like the envelope of termite mound, it not only a form but also a function to respond environment. From this complex sustainable issues, **Klaus Bollinger** agrees that Architectural design needs to incorporate complex organizational and functional requirements, and therefore constitutes a recurrent negotiation of analyzing existing and requisite conditions as well as generating and evaluating possible responses with environment [6]. Including "Water cube", "National Stadium – Bird Nest", and "Swiss re London Headquarters", there are more and more architectural projects challenge how the environmental issues associate with the multiple meanings of modern architecture.

In order to examine the feasibility of modern architecture in sustainability, the digital analytic software in three-dimension (such as Maya, Catia, and Rhino) is applied into architectural projects. Through the manipulation of morphologic algorithm of those software, architects or designers could design and clone a 「Functional Pattern」 at will, which is dealt with environmental issues nowadays. The idea of functional pattern is introduced on the book edited by Farshid Moussavi and Michel Kubo, *The Function of Ornament*. The new meaning of modern architecture is defined as a pattern design. Robert Levit also presents that the works such as LTW's Beijing, Water Cube, and Federation Square can be regarded as a product of varied cellular pattern produce a teeming articulation rather than a definite figure [7]. Although, it seems that algorithm, aggregation, pattern, and environmental issues become the most important features for those modern architects to concern, but, what is the question left behind? Robert Levit deemed that these forms appeal on a variety of symbolic level and offer an image of individuation [7]. Unlike the aboriginal architectures introduced on the book edited by Dora P.Crouch and June G. Johnson, *Traditions in Architecture*, Stilt House in Indonesia is elevated on posts and on all sides for optimal ventilation, the roof overhang provides shade [8]. Formerly, People

learn the knowledge from environment; they construct the form of architectural envelope to coexist with nature. On the contrary, the form of modern architecture's envelope presents an idea of brand or the icon of architects rather than the respect for local environment? Consequently, the study looks backward to seek a method which could modulate the requirements between environment and human's life under the theme of algorithm, aggregation, pattern, and environmental issues.

In the book, *traditions in architecture*, Dora P.Crouch and June G, Johnson clarify that the architectural morphology generated from environment is depended on the environmental condition, material, structure, and certainly, the ornament. Equally, Robert Levit interpret that the role in the constitution of architecture is a Pattern, which is one of ornament's chief incarnations. Obviously, if the study tries to approach the field of architectural morphology in environmental issues, the material, structure, and ornament would be the next themes to explore.

6. Reference

- [1]. Michael Weinstock, "Architectural Design(march/april 2008) :Matabolism and Morphology ", Artmedia Press, London.
- [2]. Achim Menges, "Architectural Design(march/april 2008) :Manufacturing Performance ", Artmedia Press, London.
- [3]. Michael Weinstock, "Architectural Design(march/april 2008) :Matabolism and Morphology ", Artmedia Press, London.
- [4]. Achim Menges, "Architectural Design(march/april 2008) :Manufacturing Performance ", Artmedia Press, London.
- [5]. George L. Legendre, "MATHEMATICAL FORM:JP's Way ", AA Publications, London.
- [6]. Klaus Bollinger, "Architectural Design(march/april 2008) :Form, Force, Performance ", Artmedia Press, London.
- [7]. Robert Levit, "Harvard Design Magazine No. 28 :Contemporary "Ornament" ", page 81, Harvard University Graduate School of Design, 2008
- [8]. Dora P.Crouch and June G, Johnson, "Traditions in Architecture: Planning and Design", page 252, Oxford University Press, Inc.