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Paper: **PHYLLOTAXIS OF THE VATICAN PIGNA**

Abstract:
In the Giardino della Pigna in the Vatican there is a giant pinecone (la pigna). The Pigna was modeled and cast during the first or second century CE.
Decorative pinecones are often used as an ornamental element. But in most cases anatomy of ornamental pinecones is different from the anatomy of the vegetative ones. Artificial pinecones are too much artificial. But the pinecone in courtyard of Vatican looks like a living thing!
In botany, phyllotaxis is the arrangement of leaves on a plant stem (from Ancient Greek *phýllon* "leaf" and *táxis* "arrangement"), as well as a mathematical discipline that went beyond studying only plants. In the arrangement of leaves on different plants one can see interesting general mathematical consistent pattern. The study of phyllotaxis has its own history. The large scientific experience was accumulated, the methodologies and the terminology were developed.
The Pigna in the courtyard of Vatican is described in mathematical terms of phyllotaxis and symmetry.

The Pigna in the courtyard of Vatican

Topic: Design Approach

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References:
[1] Roger V. Jean, "*Phyllotaxis: A Systematic Study in Plant Morphogenesis*", New York: Cambridge University Press, 1994

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Phyllotaxis of the Vatican Pigna

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Premise

In 2011 I had the honor to be invited by Mr. Soddu to the *XIV Generative Art International Conference*. Needless to say the interest of the conference itself and the interest to visit the Eternal City for the Russian, who had never visited Italy - homeland of Leonardo Pisano and Leonardo da Vinci (chronologically). For me, the interest was to find an answer to the question, troubled me long time. Once in the television show I saw a giant pinecone, located in the Giardino della Pigna in the Vatican. I was struck by the statue naturalism.
In my work the Pigna in the courtyard of Vatican is considered from the point of view of symmetry phyllotaxis and symmetry.

1. Decorative pinecones and Phyllotaxis

Decorative pinecones are often used as an ornamental element. But anatomy of ornamental pinecones in most cases is different from the anatomy of the vegetative ones. Artificial pinecones are too much artificial. But pinecone in courtyard of Vatican looks like a living thing! “Perché non parli?”

Picture 1. Perché non parli? - Why are you silent?

Let’s give some preliminary explanation.

1.1 Phyllotaxis

Phyllotaxis: from Ancient Greek *phýllon* "leaf" and *taxis* "arrangement"). Phyllotaxis is the arrangement of the leaves. Many plants with spiral leaf arrangement forming elements (leaf scales, spines, seeds, etc.) are arranged in rows distinguishable to the eye. The eye tends to connect nearest points into spirals. These spirals are called *parastichies*. Usually there are two sets of parastichies winding in different directions. On the shoots of plants contact parastichies numbers are, typically, the Fibonacci numbers. The name given in honor of Leonardo Pisano, who in 1202 published the famous *Liber abaci*. This book was first introduced a sequence of 1, 1, 2, 3, 5, ..., where each successive number is equal to the sum of the previous two. On very rare instances of plants (not taxonomic species) one can meet other series, such as 1,3,4,7,11 ... - Lucas numbers. Another example is 2, 2, 4, 6, 10..., where the numbers have a common factor greater than 1, in this case 2. Such phyllotaxis called *multijugate*. In either case, the series constructed by the rule of the Fibonacci - each subsequent equals to the sum of the two previous ones, as mentioned above.
Rising phyllotaxis - the phenomenon of change of the pair of contact parastichies due to changes in the diameter of vegetative shoots at different levels. A pair of (3:2) with increasing diameter of the cone is changed to (3:5) and then (8:5) etc. With decreasing radius on the opposite end of the cone there is the reduction of (8:5) – (3:5) – (3:2), etc. explanation. [1]

1.2 Symmetry on the plants

In mathematical terms the symmetry on the plants shoots with spiral leaf arrangement belongs to *the similarity symmetry*.

1.2.1 Similarity symmetry

In similarity symmetry, the elements are dilated by a scale factor. They retain their shape but either stretch or shrink in size. This type of symmetry is gaining attention because of its relation to fractals.

In spiral or helical symmetry, the piece exhibits a spiral or helix. In other words, there is a central vertical axis that the piece "winds" around and either toward or away from.

Ornamental pinecones usually can be described by the rotational symmetry.

1.2.2 Rotational symmetry

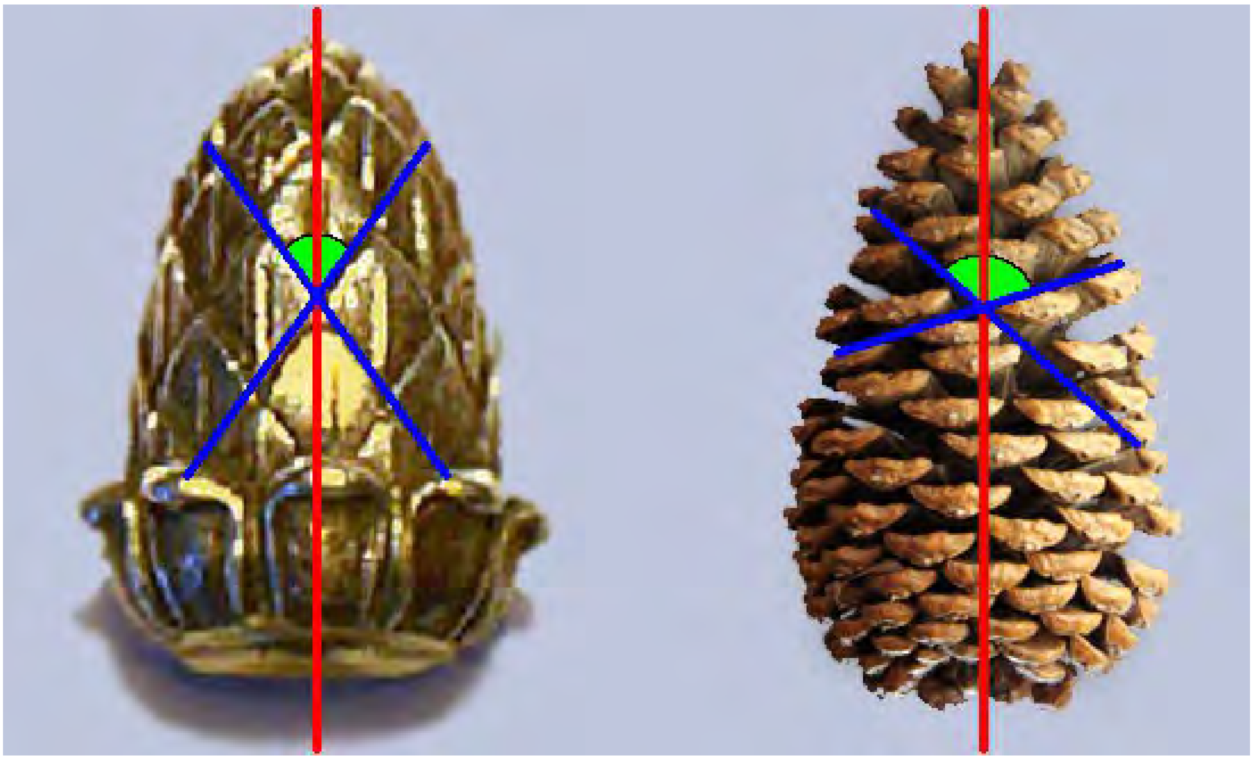
With Rotational Symmetry, the shape or figure can be rotated and it still looks the same. How many matches there are as you go once around is called the *Order*. With Rotational Symmetry, the shape or image can be rotated and it still looks the same.

In other words, one can rotate (or turn) an ornamental pinecone around a center axis by less than 360° and the pinecone appears unchanged. But with a vegetable pinecone it cannot be done.

1.3 Ornamental and vegetable pinecones

Table 1 Differences of vegetable and decorative pinecones

	Ornamental pinecone	**Vegetable pinecone**
Type of Symmetry	Rotational Symmetry	Similarity Symmetry
Number of right and left contact parastichies	The same	The different. Typically, the neighboring Fibonacci numbers.
number of contact parastichies at different levels of pinecone	The same	The different because of the rise phyllotaxis.
slope of opposite (right and left) parastichies to the direction of the pine cone axis	The same	The different
whole pinecone shape	Regular	Can be deformed.



Picture 2. Opposite (right and left) parastichies slope (blue) to the direction of the pine cone axis (red) at a work of art and at a natural pinecone.

The Pigna in the courtyard of Vatican is very much look like the natural pinecone.

Of course, when the first time I looked at the Pigna, I noticed only the deformation and different slope of opposite parastichies. I really wanted to calculate its parastichies, and such an opportunity! I do not want to belittle the artistic and historical value and other architectural treasures of the Vatican, but the Pinecone in the Giardino della Pigna represented for me the particular savor.

Our meeting took place. I behaved to her as the paparazzi. I did a lot of shots with my camera. To my great regret, I have not had the opportunity to take pictures from the part of the museum wall because of the fence. I am not professional paparazzi. The incompleteness of the picture presented difficulties for the work. I found some suitable images on the Internet, but so far I'm not sure about the absolute accuracy of my calculations carried out in Moscow.

The results of my work: Description of the morphology of the cones Vatican.

2. More detailed analysis of the anatomy of the Vatican Pigna

2.1 Number of parastichies in the pair of opposite families

Unfortunately, for the above reason I cannot vouch for the accuracy of counting the parastichies number in families of opposite parastichies of Pigna. Rough calculations yielded numbers close to 9 and 12.

Let parastichies pattern on a wide part of cone correspond to phyllotaxis (9:12) = 3 (3:4). (3 and 4 are members of the Lucas series 1, 3, 4, 7, ...). The presence of a common factor 3 relates this pattern to the category *multijugate*. In multijugate phyllotaxis, two or more botanical elements grow at the same node. Elements in a whorl (group of elements at a node) are spread evenly around the stem. Multijugate patterns look very similar to spiral.

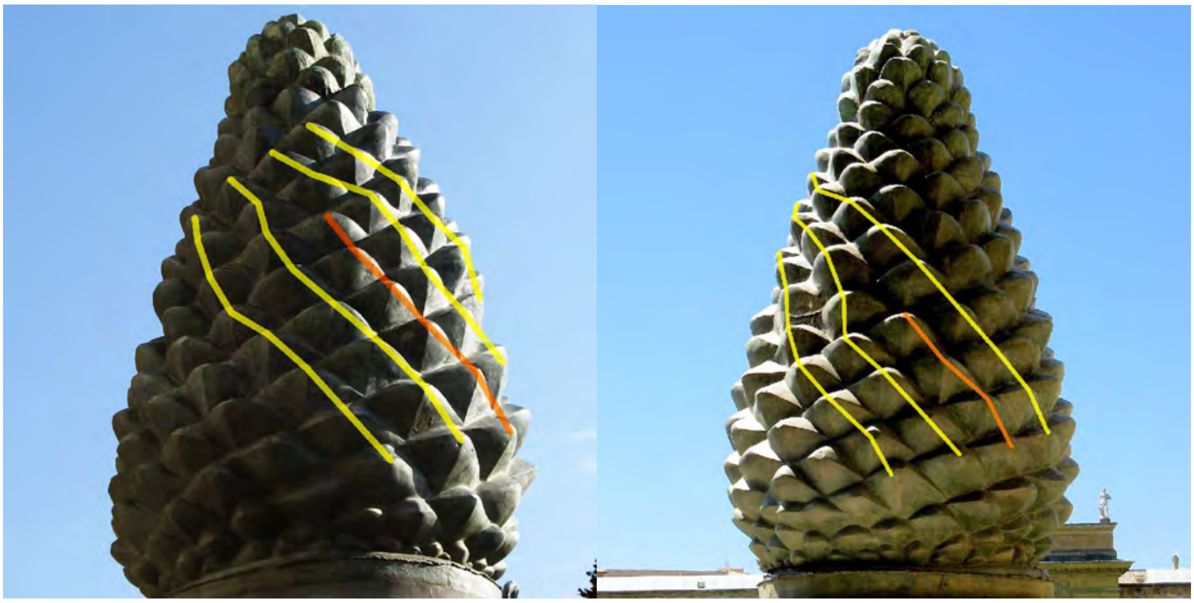
Anomalous phyllotaxis is extremely rare in the pinecones. Usually, they strictly follow the classical Fibonacci series. However, I have kept a pinecone from the city of Haifa (Israel) with phyllotaxis (6:9) = 3 (2:3). (2 and 3 are members of the Fibonacci numbers).

On the other hand, (9:12) ~ (8:13). The (8:13) is a classical Fibonacci pattern of phyllotaxis.

In any case, this question of the parastichies number in families will receive a response after an additional measurement of the Pigna.

2.2 Rising phyllotaxis

At the Vatican Pigna one can observe the phenomenon of *rising phyllotaxis*. At different levels of stem the numbers of contact parastichies are different. At the wide part of the cone the number is greater than at the narrow parts.



Side view Rear view

Picture 3. Rising phyllotaxis. Some parastichies (orange) do not reach the top. The number of parastichies on different levels of the Pigna is different.

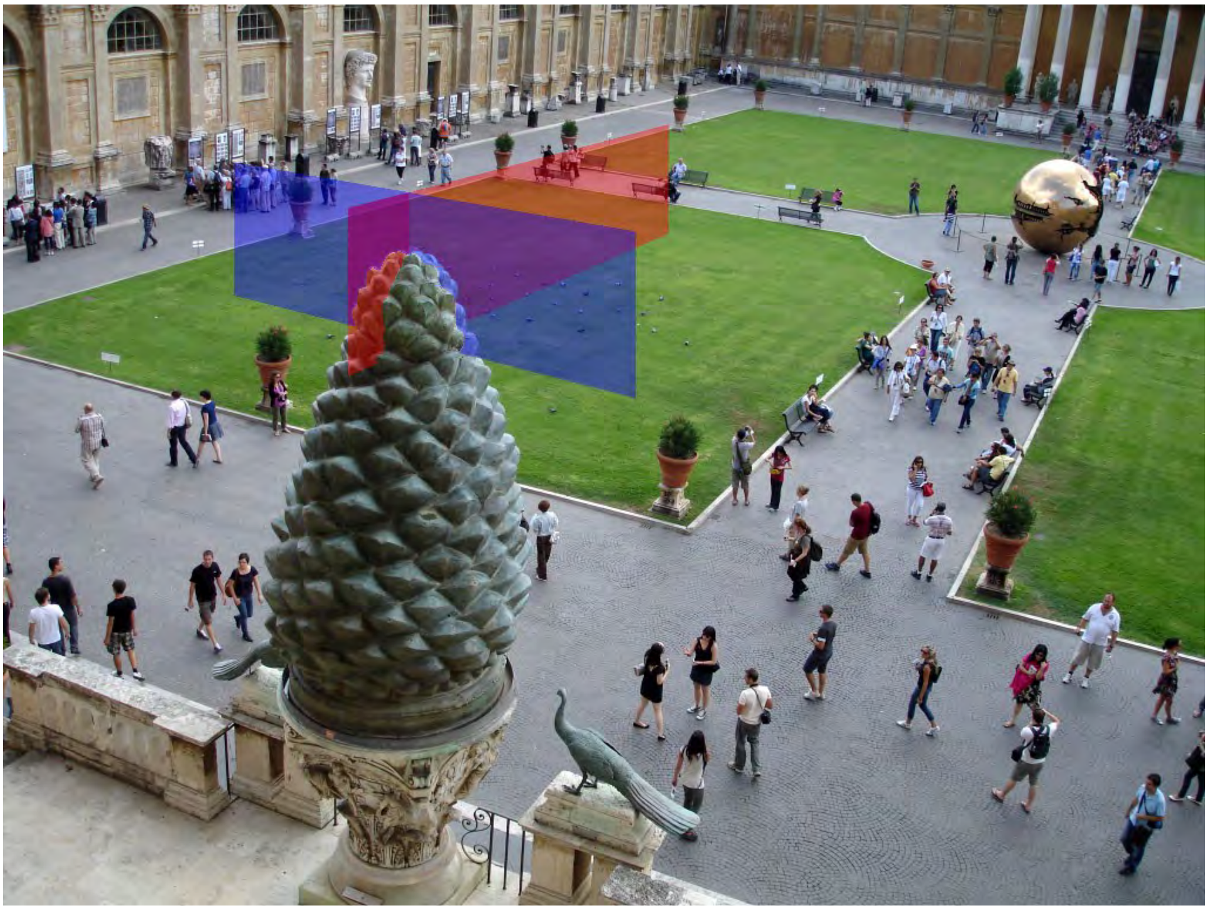
2.2 *The Pigna symmetry is ambiguous*

At the first blush, the Pigna corpus is monolithic, but in fact it divided into two pieces: the body and the cap. The boundary of these fragments is barely visible seam. The texture and color of the cap surface also differ from those on the body).

There are indications in the fall of cones during the storm, possibly as a result of lightning [2].
From the same source we learn that it is this pinecone, which, according to the author, formerly crowned the dome of the Pantheon, Vitruvius called the flower. From a mathematical phyllotactic point of view, there is no difference between pinecone and flower.

Picture 4. The Pigna corpus is divided into two pieces: the body and the cap.

You could pay no attention to the fact, but the symmetric properties of the cap and the body are qualitatively different.
The cone body has the symmetry properties of the similarity.
The cap has a mirror or bilateral symmetry properties, i.e. the cap is divisible into equal mirror halves.



Picture 5. The cap has a mirror or bilateral symmetry properties.

The top has two planes of mirror symmetry:

The median (sagittal) plane of symmetry coincides with the plane of symmetry of the courtyard.

The frontal plane is parallel to the wall of the museum behind the Pigna.

These planes are perpendicular to one another. The line of intersection coincides with the axis of the cone. Around this axis, we observe the symmetry of similarity of the body.

One can only speculate about the reasons for such eclecticism. Perhaps the original cap was destroyed and in its place was put a patch which is different from original? Restorers by this time lost the knowledge of phyllotaxis, or at least powers of observation of ancient sculptors (the Pigna was modeled and cast during the first or second century CE). The restorers completed the creation of the piece according to their notions of symmetry.

Surprisingly the body of the Pigna looks newer than the cap. It is interesting to note that the tip scales of natural vegetative pinecone are younger than the other ones, located closer to the pinecone basis.

3. Conclusion:

The Pigna is an artwork (not a copy of a natural pinecone). Otherwise, with great probability would be the pattern of the Fibonacci numbers (it is possible). Let's wait for additional measurements.
This conclusion reached in the absence of assumptions about other possible reasons of numerology. One can calculate not only parastichies, but the cone scales. Perhaps a more detailed mathematical, historical, cultural, and other analysis will propose other hypotheses.
The ancient master P. CINCIVS P. L. CALVIVS possessed powers of observation and knowledge, which later were lost. He was close to nature.

References:

[1] Roger V. Jean, "*Phyllotaxis: A Systematic Study in Plant Morphogenesis*", New York: Cambridge University Press, 1994

[2] http://www.persee.fr/web/revues/home/prescript/article/mefr_0223-4874_1881_num_1_1_6357?_Prescripts_Search_tabs1=standard& (M. Georges Lacour-Gayet, La Pinga du Vatican, p.315, 316)