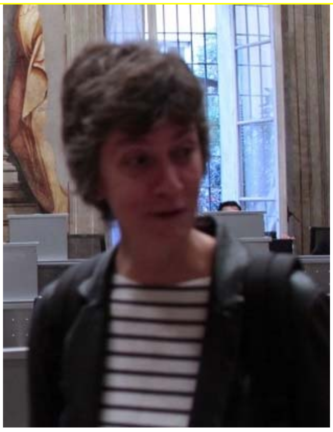

DISTANT FORCES GENERATING FORMS

Topic: Architecture

Author:
Marie-Pascale CORCUFF
France, Ecole Nationale Supérieure d'Architecture de Bretagne (ENSAB),
Groupe de Recherche sur l'Invention et l'Evolution des Formes (GRIEF)
https://www.rennes.archi.fr/recherche/grief/

Abstract

Distant forces are actually very common in the physical world : gravity, magnetism and electricity, later confounded into electromagnetism, are the most obvious ones. The absence of direct contact between interacting elements in these different forces bothered scientists till 1905, and they invented an "aether" in order to explain the transmission of them. This conception is nowadays obsolete, and is replaced by that of "field", and waves, even most recently for gravity.

Those forces may be attracting or repelling: gravity is attracting, magnetism and electricity are repelling between elements of the same sign, and attracting if their charges are of opposite sign. The magnitude of these forces depends on the distance between elements, along some well known rules.

Distance, *attraction*, *repulsion*, abstractly explored for themselves, or related to actual physical phenomena, used in static or dynamic models, are sufficient to generate numerous interesting forms, from very simple to more sophisticated ones. The models go from maps to multi-agent systems.

This paper gives me the opportunity to look at some models I showed at previous GA conferences in a different way, and to present other experiments, sometimes carried out a long time ago too, but never shown: Chladni-like patterns, "magnetic" maps, phyllotaxis, etc.

The gist of those experiments is to consider forms as the result of self-organising processes, as the emergent outcome of rules involving primarily, if not only, the *distance* between elements. They may (or may not) inspire architectural design, and this possibility is discussed in this paper too.

This paper is a tribute to Paul Coates (1945-2013) and to Frei Otto (1925-2015), whose works very much inspired me, and also to some of my students, whose ideas and expectations stimulated me in exploring new fields.

m-p.c@wanadoo.fr

Key words: *self_organisation, emergence.*

Main References:
[1] Paul Coates, "*programming.architecture*", Routledge, London and New York, 2010
[2] Frei Otto, "*Occupying and Connecting*", Axel Menges, Stuttgart/London, 2009

Distant Forces Generating Forms

Marie-Pascale Corcuff
Groupe de Recherche sur l'Invention et l'Evolution des Formes (GRIEF),
Ecole Nationale Supérieure d'Architecture de Bretagne (ENSAB),
Rennes, France
www.rennes.archi.fr/recherche/grief/
e-mail: m-p.c@wanadoo.fr, marie-pascale.corcuff@rennes.archi.fr

Abstract

Distant forces are actually very common in the physical world : gravity, magnetism and electricity, later confounded into electromagnetism, are the most obvious ones. The absence of direct contact between interacting elements in these different forces bothered scientists till 1905, and they invented an "aether" in order to explain the transmission of them. This conception is nowadays obsolete, and is replaced by that of "field", and waves, even most recently for gravity.

Those forces may be attracting or repelling: gravity is attracting, magnetism and electricity are repelling between elements of the same sign, and attracting if their charges are of opposite sign. The magnitude of these forces depends on the distance between elements, along some well known rules.

Distance, *attraction*, *repulsion*, abstractly explored for themselves, or related to actual physical phenomena, used in static or dynamic models, are sufficient to generate numerous interesting forms, from very simple to more sophisticated ones. The models go from maps to multi-agent systems.

This paper gives me the opportunity to look at some models I showed at previous GA conferences in a different way, and to present other experiments, sometimes carried out a long time ago too, but never shown: Chladni-like patterns, "magnetic" maps, phyllotaxis, etc.

The gist of those experiments is to consider forms as the result of self-organising processes, as the emergent outcome of rules involving primarily, if not only, the *distance* between elements. They may (or may not) inspire architectural design, and this possibility is discussed in this paper too.

This paper is a tribute to Paul Coates (1945-2013) and to Frei Otto (1925-2015), whose works very much inspired me, and also to some of my students, whose ideas and expectations stimulated me in exploring new fields.

Preamble: a tribute

This paper intends to be a sort of landmark for me, looking back at people that made me progress and be more confident in my work.

First of all, this is a tribute to Generative Art Conferences, to Celestino Soddu and Enrica Colabella, who have organised these marvellous meetings for twenty years now. I started to work on what I did not know was generative art a long time ago but, honestly, I felt very lonely... Few people around me were interested in this kind of research. I am not sure now but I think I heard of GA Conferences through Renato Saleri, a fellow researcher at ENSAL (Lyon). The first time I came, in 2004, it was a revelation. It was amazing to meet so many different people whose work I was interested in, whatever their field was. I participate sometimes in other conferences too now, but GA is still my favourite: it is the most open-minded, and you never know what discovery you will make, from somebody you did not imagine to have the opportunity to meet. It is the warmest too, and I do not refer to the climate. Milan in December can be very cold... No, I refer to the people, and to the diligent though friendly organisation.

Among all the people I happened to meet with benefits, and that I cannot all enumerate, I want to evoke Paul S. Coates, professor and researcher at Centre for Environment & Computing in Architecture (CECA), University of East London, who unfortunately died five years ago. Beyond what he brought to me intellectually, and is recorded in his essential book [1], I want to recall the person he was. I met him the first time I participated in GA in 2004, but he was a recurrent participant himself, since nearly the first conference. This first encounter was not mild. He could be abrasive in questioning my work, and I had a hard time understanding his perfect English (himself was unable to speak one word of French, though he owned a holiday house in centre Brittany (Bretagne), not far from where I live...). But I learned to appreciate him, his congenial sense of humour, even if I was unable to catch all his jokes. He was a very generous teacher, and was often accompanied by some of his students, beyond his collaborators, and always valorised their work. He complained about the difficulties some of his foreigner students had to get their visas. Luckily, through his too early death, he had not to know the Brexit...

I did not know Frei Otto personally, obviously, and he apparently does not belong to this world of generative art. This architect is well known for his lightweight structures, but I discovered in 2010 a rather different aspect of his research through his very precious small book [2], where he questions many ways in which forms and configurations emerge, are generated through simple rules and physical laws.

Last, I want to thank my students. The school of architecture where I teach is a small one (around 500 students in all, for 5 years of schooling), and very few students choose my seminar, which is an option among a lot of other ones, some very attractive, and certainly more professionally oriented. Having to learn coding is a repellent for the majority, but those bold (or thoughtless) enough to take the plunge generally do not regret it, and tell me so sometimes long afterwards. Anyway I must say I benefit a lot from them, first because you never completely understand what you are doing unless you have to teach it, and also because their fields of interest expand mine.

Introduction

Distance is at the core of our conception of space. A metric space is a set defined by the type of distance one defines between members of it. The Euclidean space is the one we are convinced to live in, a 3 dimensions space as I recalled in my paper of GA 2005 [3]. It is not before the 19^{th} century that mathematicians invented non-Euclidean geometry, in which distance between two points is not the length of a straight line drawn from one to the other. Actually, would Pythagoras and Euclid have lived on a much smaller planet, say the Little Prince's planet, they would have conceived a very different geometry, at least for figures drawn on the surface of their planet...

All the processes investigated here deal only with *distance*: distance between elements, distance from elements towards some specific ones. The first part deals with static processes, where distance, distant forces, modify elements but without moving them, and the second part deals with dynamic processes, where those forces make elements move, whether they where static at first, or already moving.

1. Static models: maps

What I call a map is, referring to geographic maps, a plane (a 2D space), on which some informations are put in a visible way (colour). More precisely, I consider that a map is a bitmap, a set of pixels, where the colour of each pixel is determined by any process you can imagine. For instance, the pictures of Mandelbrot sets are maps, in which the colour of each pixel (x,y) is determined by the way the iterated function $f_c(z)=z^2+c$ (where c=x+iy) evolves from z=0.

A map is in this case a sampling: the complex place, as well as the real plane, being continuous, while a bitmap is discrete. The coordinates of pixels correspond to only a few of the points of the complex plane, and when you "zoom" in, you actually sample more densely a smaller part of the plane, but it is still sampling.

Actually the way we draw geographic maps today is always through sampling. A digital elevation model is a map where the altitude of some points (but obviously not all of them) is recorded. And you can interpolate, because there is no chance at all that between two points the altitude will change dramatically. The same goes for other data: direction and strength of the wind, temperature, and so on.

1.1 Distance maps

I showed my first distance maps in 2005 [3], and as I recalled in this paper the idea came from "medial axes" [4]. A distance map is a set of pixels in which the colour of each pixel depends on its *distance* from the nearest element of a given set of points (called sites). There are diverse ways of translating this distance into colour. The first is to give the pixel a colour arbitrarily affected to the nearest site, which is the classical rendering of Voronoi diagrams (which distance maps are, actually); the second one is to give the pixel a level of grey proportional to that minimal distance, which leads obviously to a 3D interpretation by translating the levels of grey into altitudes in a mesh; and then you can give the pixel a level of grey proportional to the sine of the minimal distance.

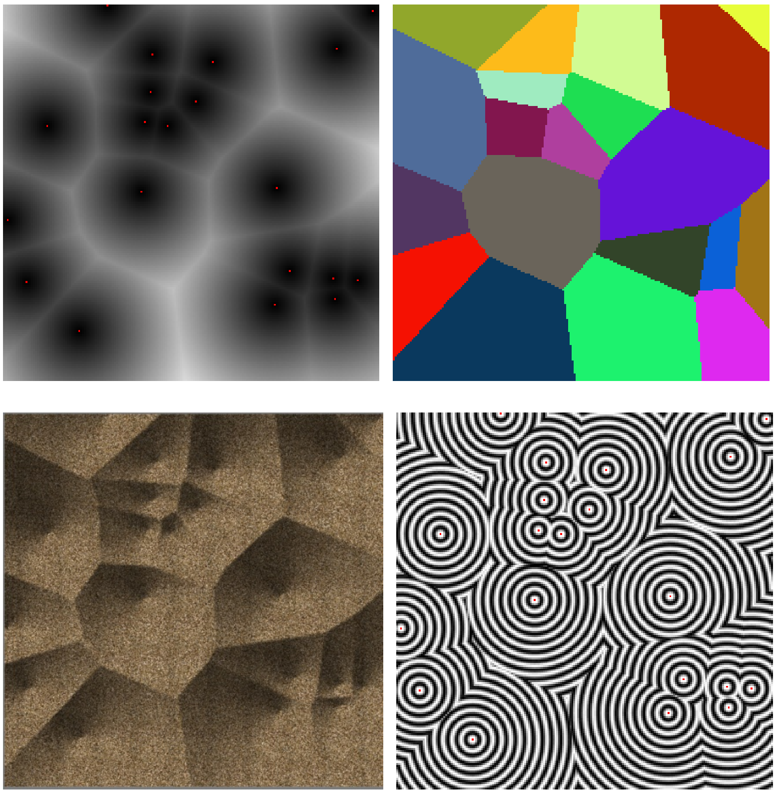

Fig. 1: four renderings of distance maps for the same set of sites

The relief obtained in the mesh representation may be interpreted as the one sand leaking from a box through holes drilled in its bottom (at the position of the sites) would produce. As sand has a characteristic slope, at least ideal sand would produce concave cones with their summit at the sites, and crest lines resulting from the intersection of these different cones; sand piling under the box would produce the inverted relief of intersecting convex cones.

I never bothered to realise this experiment with sand, and was very excited to discover that Frei Otto described the very same experiment in [2]. I developed the consequences of this in 2011 [5]. But recently a further step has been taken: while Frei Otto seemed to consider this experiment more as a thought experiment (it is not sure he actually did it: there are sketches of the proposed device, but no photographs), the research one of my student did on granular matter revealed that some people, not only made a "real life" experiment, but managed to solidify the surface of the relief in order to create an architectural model...

It was interesting to see that what was a purely intellectual preoccupation could actually be useful for design, even though I think that doing it digitally would be much better, because modifying the parameters of the experiment (the location of the sites/holes, particularly) would be much easier. And what is the purpose of such an experiment if you can not easily change the conditions of it?

The last representation consists in giving the pixel a level of grey proportional to the sine of the distance. I made those pictures first because the reference I had was about Japanese Zen gardens, and that monks most often rake the gravel around the stones (which were the so-called sites in that model). But I discovered again later that it could be linked to Frei Otto's proposal of a thought experiment with seeds that would expand in a concentric way, year after year. The circles drawn by

Otto were nicely depicted by the concentric waves of the sine-distance maps [5]. One could even simulate the different speeds in which seeds would grow.

I wanted to experiment further for this paper by applying a non-euclidean distance. The space is the Poincaré disk model, an open disk, where "straight" lines are, first, the diameters of the disk, and, if the two points are not on a diameter, the arcs of circles orthogonal to the boundary of the disk. It demands some geometry in order to draw those lines, but it is not undoable. The distance between two points A and B is then ln(PB.QA/PA.QB) where P and Q are the intersection of the "line" (which is actually a circle) with the circular boundary. Again, not simple, but not undoable... It is then possible to make distance maps for this distance, and compare them to euclidean distance maps for the same sites.

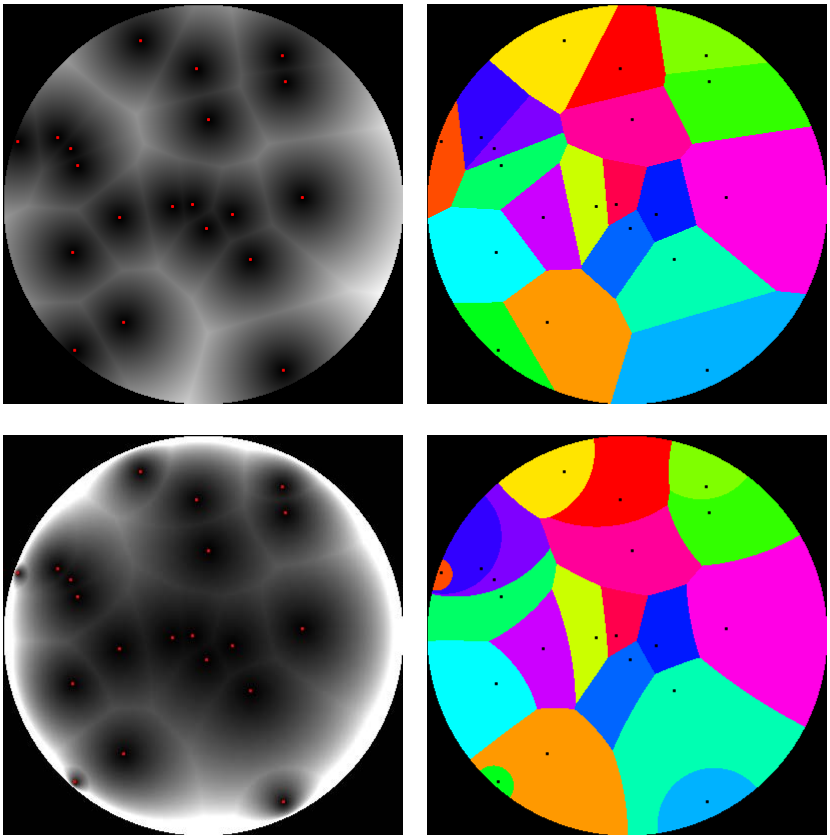

Fig. 2: distance maps with the same 20 sites; above: euclidean distance; below: non-euclidean distance

1.2 Wave maps

The sine-distance maps lead to another idea: those waves could be those happening from throwing rocks in a pool of water at the location of the sites. But weirdly those waves timidly interrupted themselves as soon as they encountered another one... That is not what happens with real waves, where altitudes (calculated from the surface of the still water, and can then be positive or negative) add themselves in every part of the field.

Now, considering those pixels for which the resulting altitude is zero or close to zero, and exhibiting them with a contrasting colour, we have what Chladni did using acoustic waves instead of water waves: the sand he poured on his metallic plate was ejected from parts of the plate which were vibrating, and assembled themselves where it was still, the nodal lines. It has to do also, maybe more so, with cymatics. Those references were well exposed at the last session of Generative Art [6].

The principle of wave maps is very simple: you calculate for each pixel the sine (or the cosine) of the *distance* (actually $d*l_w/2\pi$, l_w being a wavelength arbitrarily chosen) to each site, and you add those values. With only one site, and if you neglect the boundaries (you consider your screen as a window on an infinite universe), it is very disappointing, you only get concentric circles... But it suffices to introduce the reflection of the wave on the boundaries (as it actually happens for a rock you throw in a small basin), which is done by introducing virtual sites, symmetric from the initial one along each side of the boundary, for the patterns to actually look like Chladni's ones.

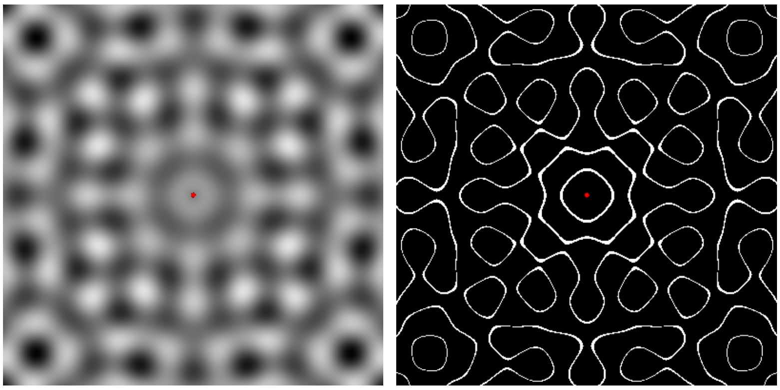

Fig. 3: wave map and Chladni-like pattern

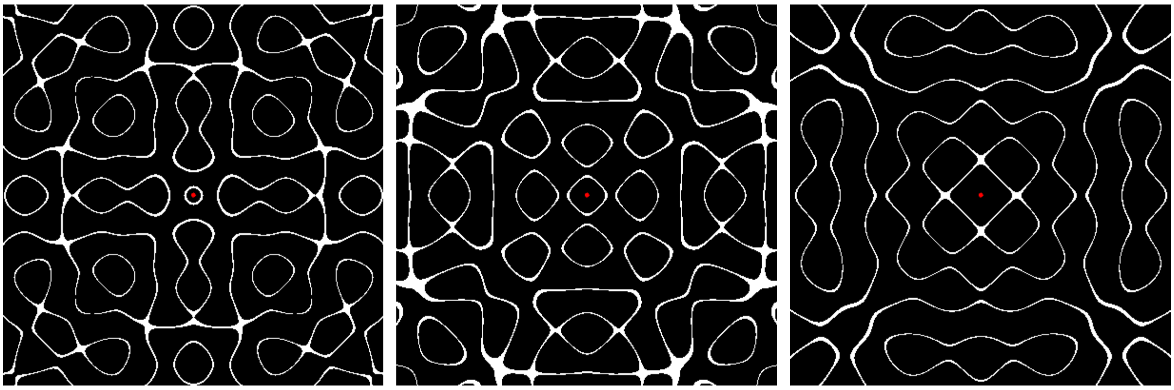

Fig. 4: patterns for a square plate with the fixed point at the centre

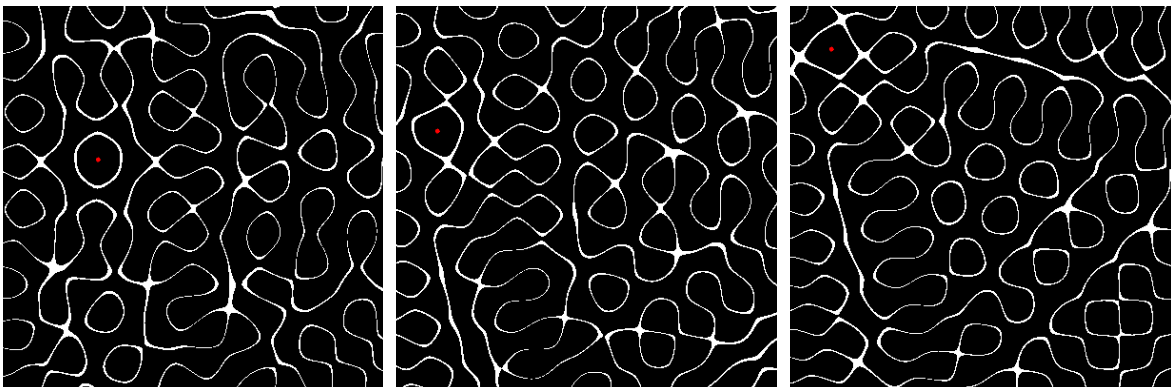

Fig. 5: patterns for a square plate with displaced fixed point

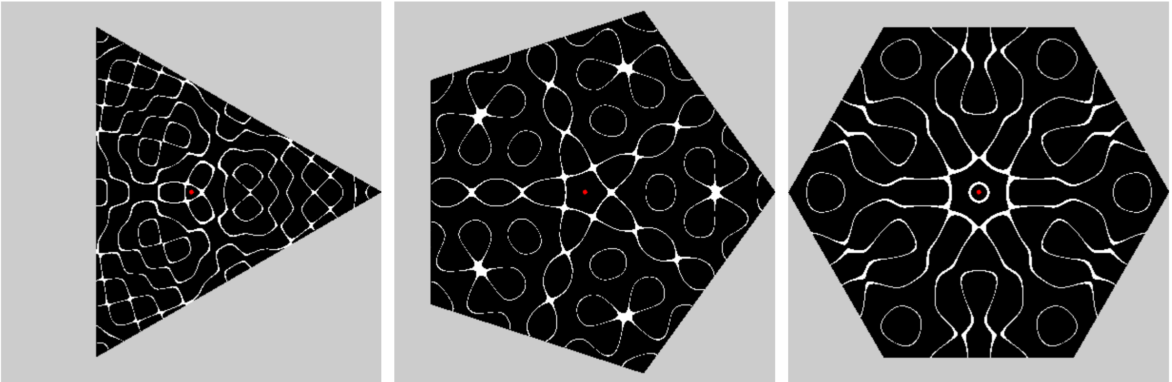

Fig. 6: patterns for other polygonal plates

One should not be fooled by the resemblance though, which is why I call these patterns “Chladni-like patterns”. The physics involved in proper Chladni’s patterns is more complicated, the natural frequency and the normal modes of the material of the plate are crucial, and I am not sure the edges of the plate are reflecting the acoustic waves.

Choosing the sine or the cosine means a different role for the initial site. Though being a map, and so a static figure, such a pattern is a sort of instant photography of a dynamic one: the sinusoidal wave oscillates vertically, the nodal points (those that have a zero altitude) being the only ones that do not move. As sin(0)=1 and cos(0)=0, using the sine function is more relevant for the water wave analogy (because where the rock is thrown there is obviously some displacement of water), while the cosine function is more appropriate for referring to Chladni’s figures, where the centre of the plaque is fixed.

I did those experiments many years ago, just for the pleasure of it, but as [6] suggests, those patterns could be useful in architecture or design in general.

1.3 Magnetic maps

The idea of working with magnetic fields was that of one of my students, David Berger. I was very sceptical at first, I did not see what it would lead to, but I was wrong as we will see. David was first attracted (without joking) to this topics through the well-known pictures of magnetic lines of force of a bar magnet shown by iron fillings, an experiment probably any child has done at school.

Our first experiment was to translate this process into an algorithmic one. A dipole is represented by two sites, a “North Pole” and a “South Pole”. The iron fillings are represented by little segments randomly distributed. They do not move, but their orientation results from the attraction-repulsion created by the dipole. In each point, you can calculate the vector resulting from the attracting force of one pole, and the repelling force of the other. Those forces are inversely proportional to the square of the distance. So the *distance* of each point from the poles suffices to define its specific orientation.

After having verified that our model was consistent with the physical one, we extended it by involving two dipoles, and see what happened with the segments. Interesting configurations were obtained, and we expanded our research in two directions.

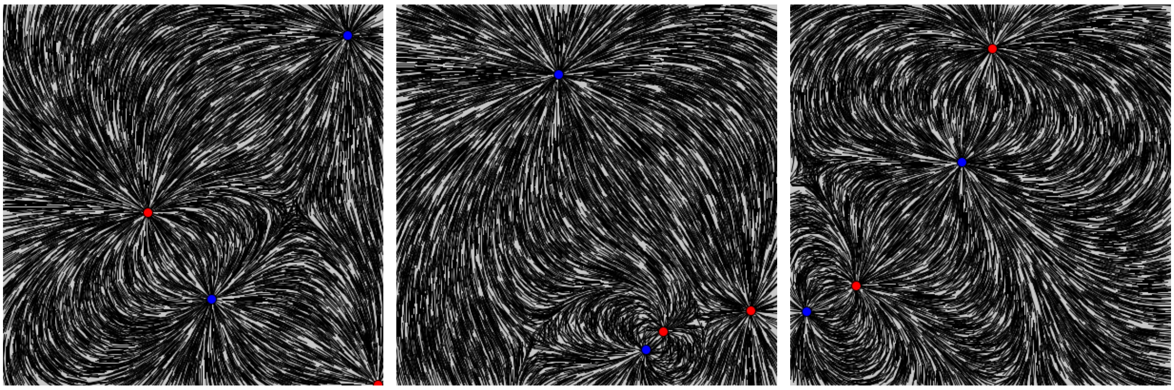

Fig. 7: two dipoles acting on segments

The first one was to make a proper map, similar to distance maps. In each pixel we calculate the orientation, and translate it into a level of grey. The result was fine, a little hypnotic when rotating the dipoles about their middle point.

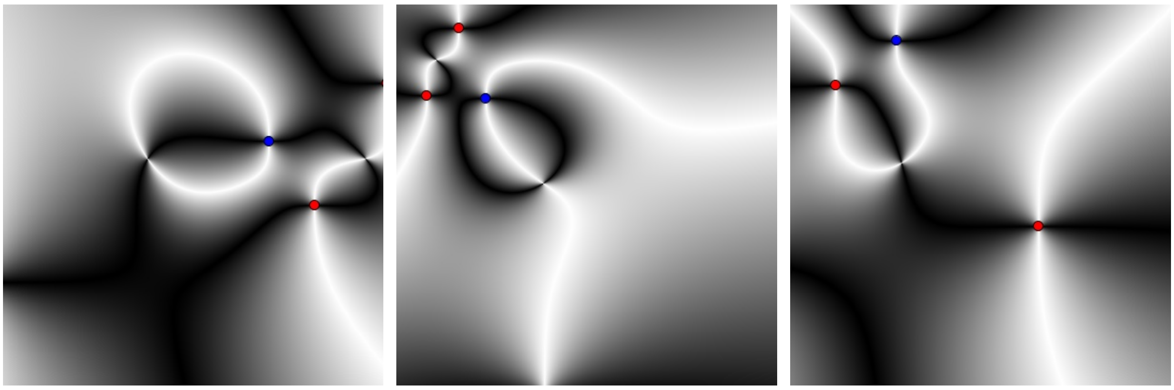

Fig. 8: angle maps for two dipoles

But David had another idea, which consisted in starting from one of the repelling poles in a random direction, and going step by step in the direction given by the orientation proper to the attained point, towards an attracting pole. Here again, the strength is not taken into account, all the steps are equal. The results were fine enough to let him interpret them as a structure, which he built in cardboard, and also integrated in a CAO model.

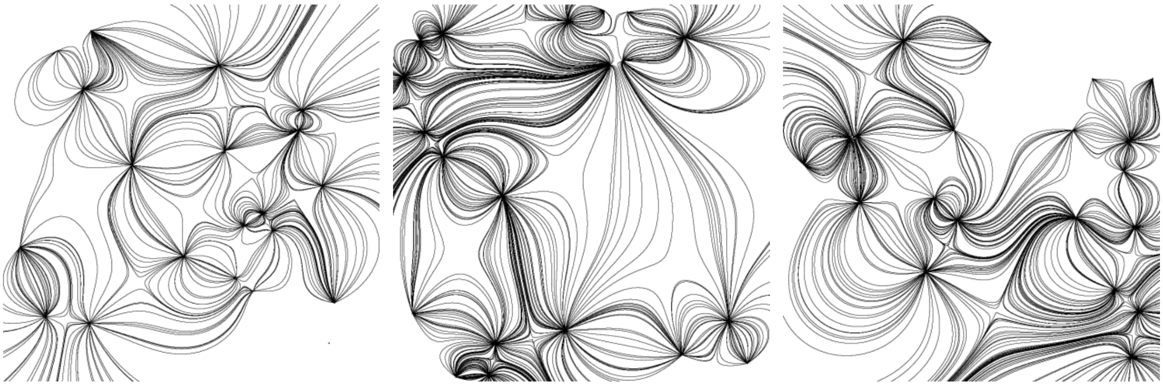

Fig. 9: trajectories in a magnetic field with 10 dipoles

All our experiments stuck to this idea that the magnetic field is a field of orientations, we did not involve the strength of the resulting force.

2. Dynamic models: emergent patterns

The previous models, generalised as "maps", may have involved distant forces, as in the magnetic field, or even when we interpret distance maps as sand flowing through holes under the force of gravity. But those models were not actually dynamic, nothing was really moving.

Here we shall see models where agents are actually moving through attracting and repelling forces, towards or from each other, or/and towards or from particular sites, and in all these models, *distance* is the key parameter. A great difference between these models and the previous ones is the type of space involved. Maps lay in the discrete space of pixels, while the models we shall see now involve a continuous space, even if we visualise their behaviour through pixels.

2.1 Optimised distributions

The first model we can think of, involving only repelling forces, consists in launching a number of particles, and enjoin them to repel each other. If there is no boundary, very soon all particles will disappear from your window, bound to an infinite journey through the universe! We deal with that in two ways: either we install a boundary, that particles cannot pass, or we consider a torical (also called periodical) topology, in which particles that disappear at one side reappear on the opposite one: a finite space without boundaries.

Frei Otto [2] experimented with small rod magnets, all oriented in the same way, floating on water (the rods are driven through a little swimmer, probably a small piece of polystyrene or cork). The distant force here is then the magnetic one. The edges of the basin obviously are dead ends, and indeed a majority of the rods get stuck to them. I showed in [5] some digital experiments simulating these devices.

I encountered the same topics in [1]. Paul Coates experimented with repelling forces among “turtles” (the name of the agents in the Netlogo language he uses), in a square with torical topology.

But what does “repel” mean in these models? In Otto's physical model, each rod is repelled by all the others, with a strength inversely proportional to the square of the distance, at least if one does not take friction into account. In Coates' digital model, each turtle takes “one step back” from the closest turtle, whatever this minimal distance is.

The result is always the same, and tells a lot of the nature of the 2D space: the particles distribute themselves along an emergent triangular lattice, which is the one that optimises the space, i. e. that permits to each particle to be the more distant from any one else. The Voronoi cells, dual from this triangular distribution, are hexagonal. Those triangular and hexagonal grids are not absolutely regular, because there is a conflict between the shape of the space (even when it is “limitless”, but finite, as in the torical topology) and a hexagonal tessellation, as we discussed in [7]. Regular hexagons tile the 2D space, but only when it is infinite!

Distance maps are used here to show more clearly the tessellations produced by the distributions.

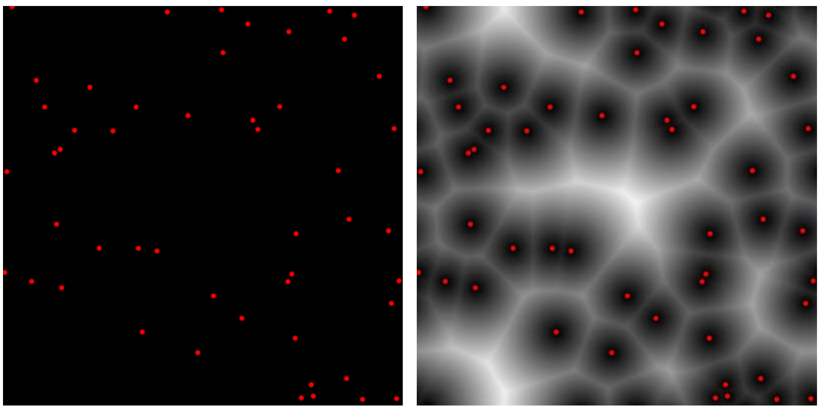

Fig. 10: initial random distribution of particles

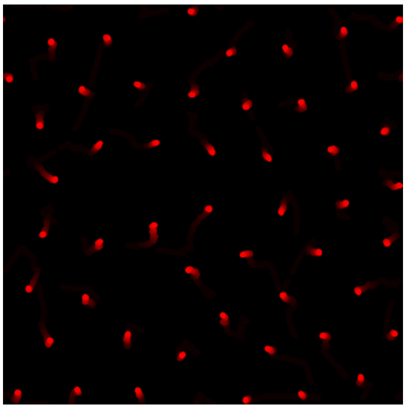

Fig. 11: wriggling particles repelling each other

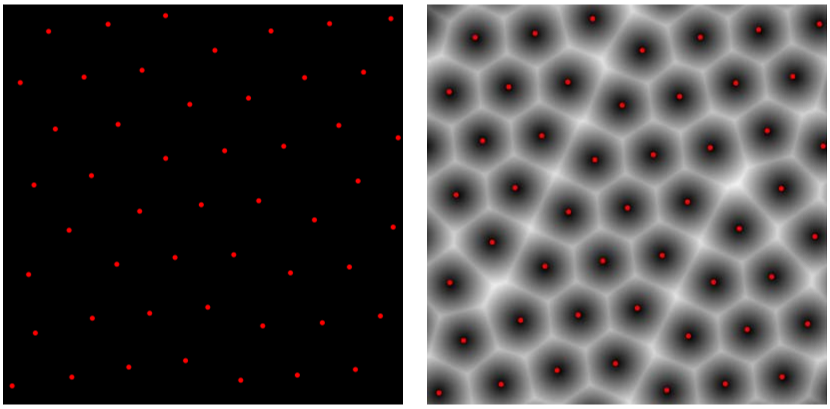

Fig. 12: resulting optimised distribution of particles

I also experimented with repelling forces as in Coates's model, but with the use of the Manhattan distance (where distance from (xa,ya) to (xb,yb) is |xb-xa|+|yb-ya|). Results show Voronoi cells which are no more mostly hexagonal, but mostly squares, with some interesting configurations as octagonal cells. It is interesting how those results, without being orthogonal grids in whole, look a lot like urban plans.

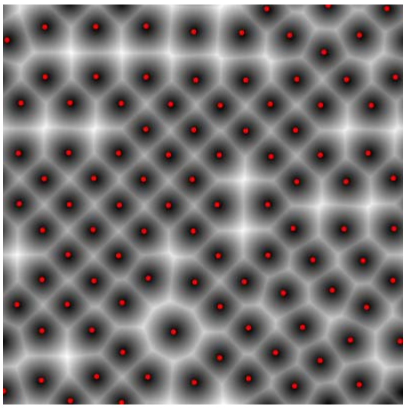

Fig. 13: optimised distribution of particles under Manhattan distance

2.2 Circles

This example is directly drawn from Coates' book [1]. It is very easy to draw a circle, by its equation, either the Cartesian or the parametric one. But by using only attracting and repelling forces, one can also obtain a circle, as an emergent pattern.

The algorithm is very simple: a centre is given, and a radius, and particles randomly distributed are affected by an attracting force towards the centre if their distance is superior to the radius, and repelled by it if it is superior. A circle emerges, but it is a little uneven. So another repelling force is introduced, this time between particles themselves: each particle is repelled by its closest neighbour.

This time the circle is more regular, and you get a "bonus" result: is the radius is small, and the repelling force between particles strong, all particles cannot fit in the circle, so they are ejected. But most of them do not go to random places, but form other circles instead. Which is quite surprising!

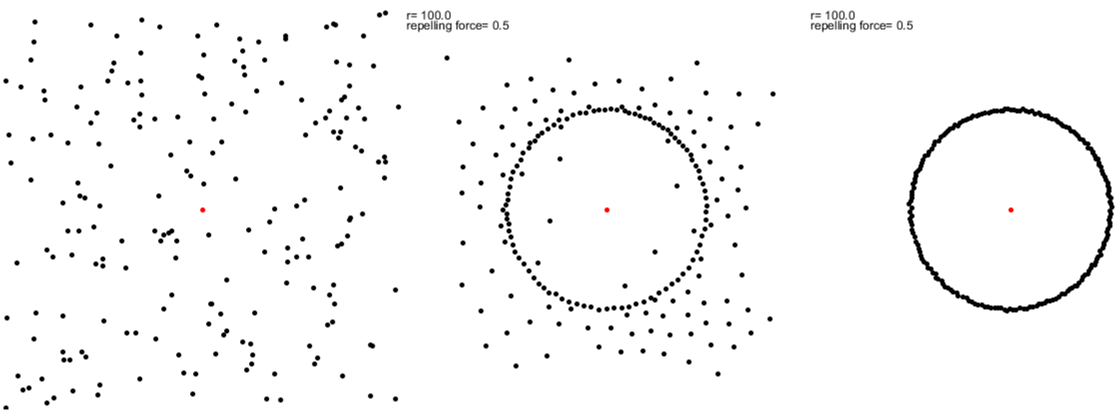

Fig. 14: emergence of a circle

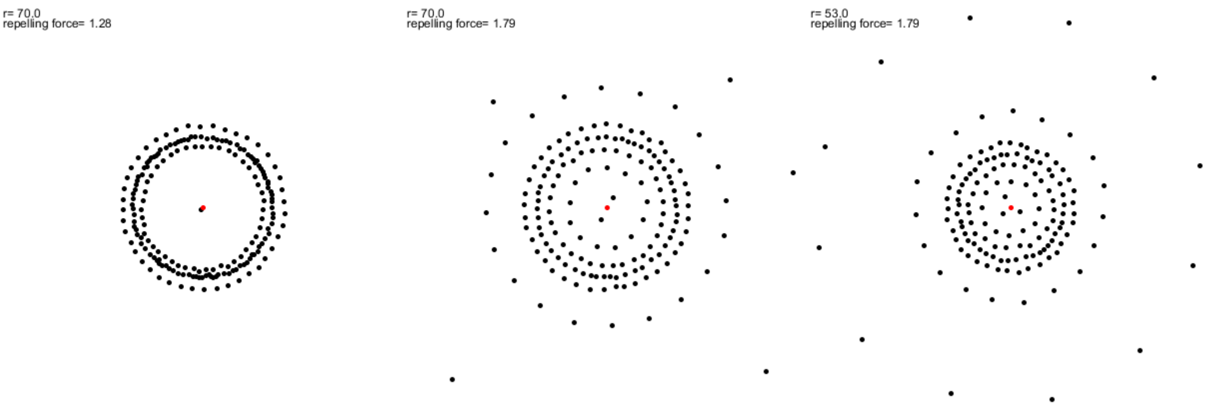

Fig. 15: emergence of additional circles

2.2 Spirals

This last model is not inspired by Otto nor Coates, but is the algorithmic interpretation of an experiment made by S. Douady and Y. Couder in order to explain the emergence of spirals in plants [8].

The apparatus used by these physicists is simple: drops of a ferrofluid fall at the centre of a dish filled with silicone oil and placed in a vertical magnetic field. So the drops are repelled from the centre. The centre of the dish has a small bump, so that *a priori* the drops fall in a random direction, and continue in that direction until they reach the edge of the dish (where they fall into a ditch).

But the drops are repelled from each other as well. So that a first emergence appears: the direction of each new drop will depend on the repelling effect of one or more of the previous ones. A steady regime of divergence (the angle between the direction of two successive drops) establishes itself, which leads to the fact that the drops are points of a spiral.

Depending on a certain parameter G, corresponding to advection, this divergence angle may change. $G = v_0T/r_0$, where v_0 is the initial velocity of the drops, T is the periodicity of the fall of the drops, and r_0 is the initial distance of the drops to the centre.

For at least some divergence angles, a second emergence occurs, which is the one we admire in sunflowers and many other phyllotactic spirals: what we perceive in them is not the generative spiral itself, but secondary spirals, which happen to be in Fibonacci numbers.

Douady and Couder have transposed their physical experiment into a numerical one involving a repulsive energy from previous particle, determining the “place of birth” of the new particle. I chose to stick more closely to the physical model, by placing each new particle at the centre, and determining its moving direction through the repelling forces of all the previous particles. The first few particles (3 in the case of the illustrations) have got a random direction. Rapidly, a steady divergence angle establishes itself. The velocity of all particles is the same, and is fixed at the start. It is the crucial parameter in this model. I found that a velocity of 1.385 lead to a divergence angle of $2\pi(1-\varphi) \approx 137.5°$ where $\varphi = (\sqrt{5}-1)/2$, the golden number, and so to Fibonacci spirals.

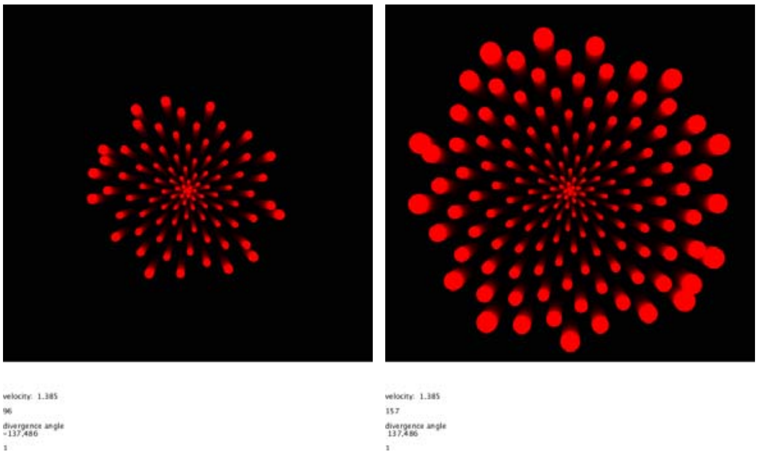

Fig. 16: rendering of the process showing particles moving straight from the centre

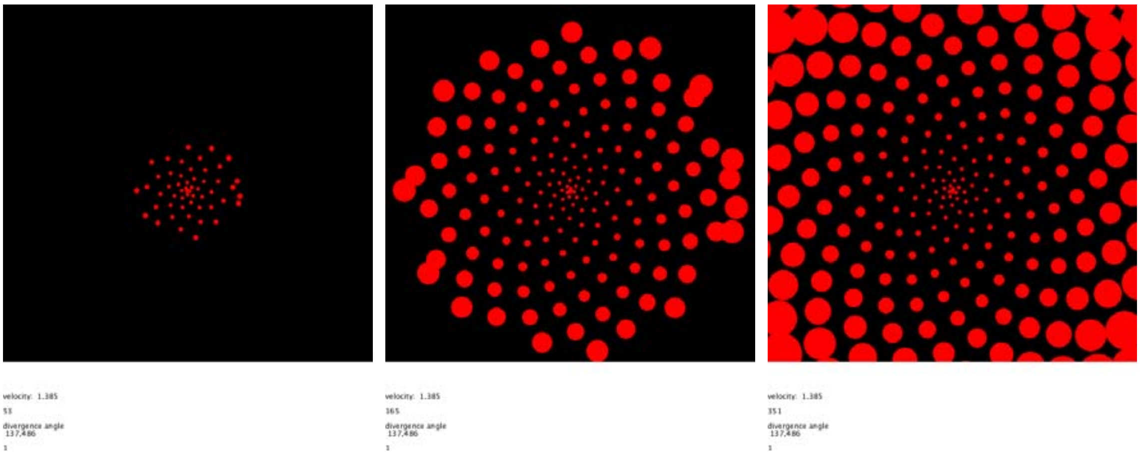

Fig. 17: process with velocity = 1.385 leading to divergence angle = 137.49°

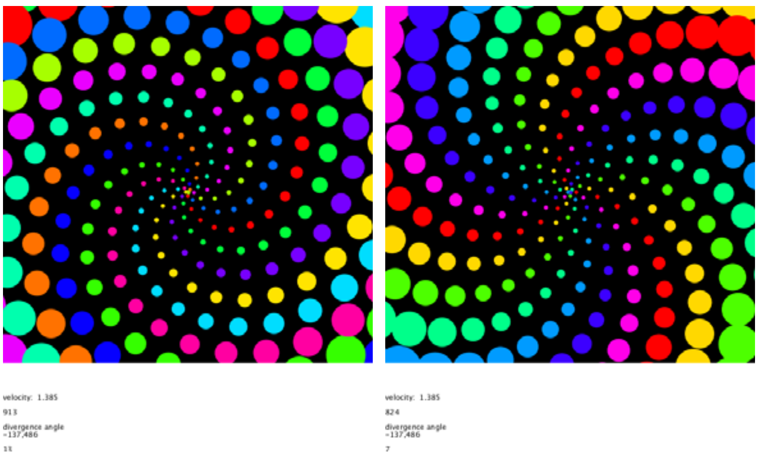

Fig. 18: emerging Fibonacci spirals (13 and 21)

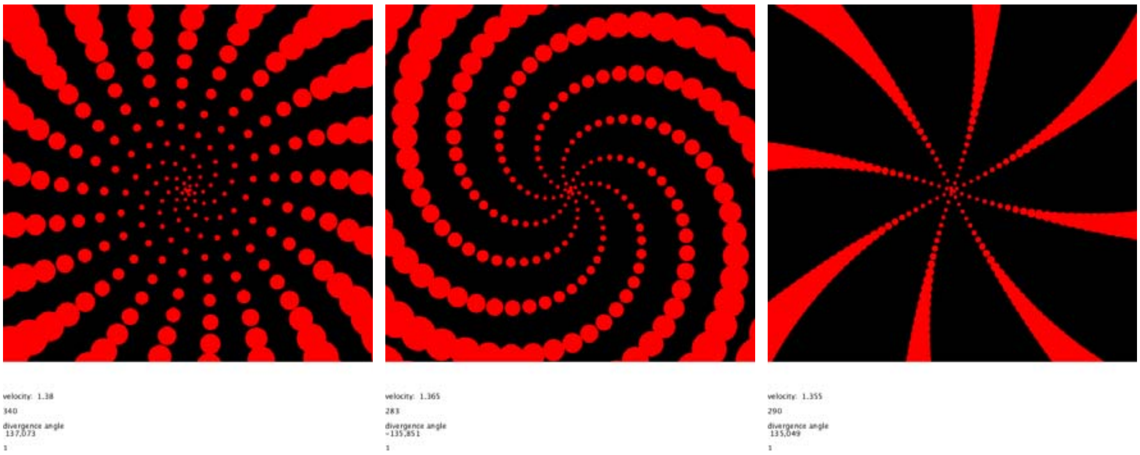

Fig. 19: other values of velocity leading to other divergence angles

These experiments will be prolonged. The question of transposing this process into a designing one is still at stake.

To finish, I would like to show what one should never show: the consequence of a stupid error... I wanted to "kill" the particles when they get to the boundary, but I forgot to remove not only their positions, but also their displacement vectors. The process became a little wild, and produced some weird, but interesting results... That is called serendipity!

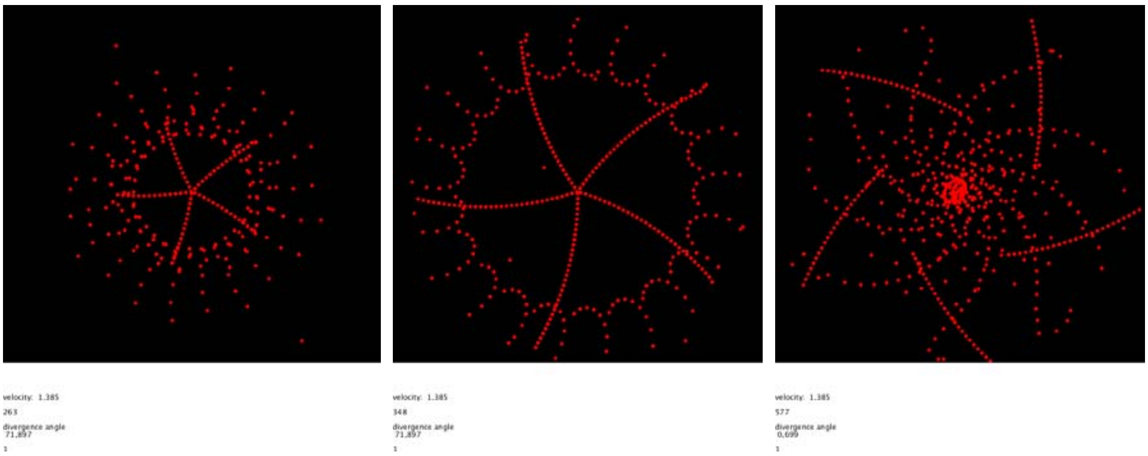

Fig. 20: surprising results due to a little error in the code!

Conclusion

This paper did not aspire to be exhaustive regarding models involving distance, and distant repelling and attracting forces. Swarming, for instance, is a very elaborate model where distance is at stake too. But, looking at my previous work, putting aside models like fractals, or cellular automata, I realised that a lot of models I had explored had that notion of distance at their core, and found it would be interesting to assemble and confront them.

References

[1] Paul Coates, "*programming.architecture*", Routledge, London and New York, 2010
[2] Frei Otto, "*Occupying and Connecting*", Axel Menges, Stuttgart/London, 2009
[3] Marie-Pascale Corcuff, "Generativity and the Question of Space", 8th Generative Art Conference, 2005
[4] G.J. van Tonder, M.J. Lyons, Y. Ejima, "Visual Structure of a Japanese Zen Garden", *Nature*, 419:359-360
[5] Marie-Pascale Corcuff, "Territories, Boundaries and Connections", 14th Generative Art Conference, 2011
[6] Varlik Yucel, Ilke Yildan, "Form Follows Algorithm: Differentiation of Chladni Patterns through Mathematical Functions in Processing", 20th Generative Art Conference, 2017
[7] Marie-Pascale Corcuff, Renaud Munch, Gwenn Rannou, "Randomness, (Dis)order and Generativity", 17th Generative Art Conference, 2014
[8] S. Douady, Y. Couder, "Phyllotaxis as a Physical Self-Organized Growth Process", Physical Review Letters, Volume 68, Numnber 13, 30 March 1992