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Video Installation: **THEOREM 8**

Topic: ***computational cinema***

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Abstract:
Theorem 8 is a 3-channel video projection installation exploring the intersection of art and mathematics. It is made using a custom software that decomposes every frame in a movie using a fixed database of frames from another movie.

The project is based on the mathematical concept of orthogonal decomposition. The idea is to select a fixed set of frames from one movie and then allow every frame in another movie to make a shadow projection onto each of those frames. (The concept of a shadow is based on the mathematical idea of a vector projection). Every frame in the second movie is decomposed onto those various changing shadows.

By mixing the different shadow projections, it is possible to achieve an approximate reconstruction of the current frame in the first movie.

The 3-channel installation setup shows the original movie, Godard's *Alphaville*, and its reconstruction in the center image. Its shadow projection onto 72 frames is shown on the left and the right screens. The frames are taken from the (never completed) film *Witch's Cradle*, directed by Maya Deren in collaboration with Marcel Duchamp.

Technical details as well as a non-technical video documentation, can be found here:

http://theorem8.concept-script.com

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Theorem 8

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Abstract
This paper explains the concept and technical process of the work *Theorem 8*. This project is a 3-channel video projection installation exploring the intersection of art and mathematics, specially the concept of orthogonal decomposition.

1. Introduction

Theorem 8 is a 3-channel video projection installation exploring the intersection of art and mathematics. It is made using a custom software that decomposes every frame in a movie using a fixed database of frames from another movie. The decomposition is based on the mathematical concept of orthogonal decomposition. The title, *Theorem 8*, refers to the orthogonal decomposition theorem, which concerns the idea of an object in a higher-dimensional space projecting shadows onto a lower dimensional space.

2. Technical description

This technical description assumes some basic familiarity with elementary ideas in linear algebra, such as the concepts of a vector, a vector space, and linear independence. It is meant for the technically inclined reader who wishes a more in-depth account of the artistic procedure employed in the making of *Theorem 8*. The first section explains the basic terminology, while the second section details the actual procedure used to generate the images in the installation.

2.1 Background Notions

A frame is a set of n x m pixels, where n and m are the width and height of the image. A movie is a sequence of frames.

Associated with every frame f in movie M is a function $B_f(x,y)$ whose inputs are the screen coordinates of a specific pixel and whose output is a floating point number in the range (0,1), the “brightness” or “value” of that pixel.

A frame f can also be described as a vector in $R^{n \times m}$ whose (y * m + x)th component is given by $B_f(x,y)$. In other words, the vector is an array containing the values of every pixel in f. They are sequentially ordered, so that the values of the pixels in the first row are followed by those of the pixels in the second row, and so on.

We take the set F of all frames of n x m pixels as a vector space, equipped with the two basic operations of vector addition and scalar multiplication. The operation of vector addition takes two vectors and outputs a new vector formed by adding corresponding entries in each input vector. The operation of scalar multiplication takes a floating-point number and a vector and outputs a new vector formed by multiplying each component of the input vector by the input number.

If F is to be a proper a vector space, it must be closed under the operations of vector addition and scalar multiplication. Frames must be allowed to contain values in the range $(-\infty, +\infty)$. This requirement poses a problem for any pixel-based visualization. In particular, how will negative values be visualized, since there is no such thing as a pixel with negative brightness? The solution adopted here, for visualization purposes only, is that any number > 1 will be set to 1 and any number < 0 to 0. All computations will otherwise be performed on the actual values. Recognizing that this solution is inelegant, the artist experimented with other possibilities, but the current option gives the most arresting visual result and so was selected because of aesthetic considerations.

We endow vector space F with a dot product, an operation whose inputs are two vectors and whose output is a single number. Given two input vectors, its dot product is obtained by multiplying every pair of corresponding entries and adding the results together. The expression $<u_1, u_2>$ will denote the dot product of vectors u_1 and u_2.

The concept of a dot product is important, because it can be used to define what it means for two vectors to be perpendicular to one another. Two vectors u_1 and u_2 in some vector space V are orthogonal (or perpendicular) if and only if $<u_1, u_2> = 0$. The "size" or "length" of a vector u, written $\| u \|$, is given by

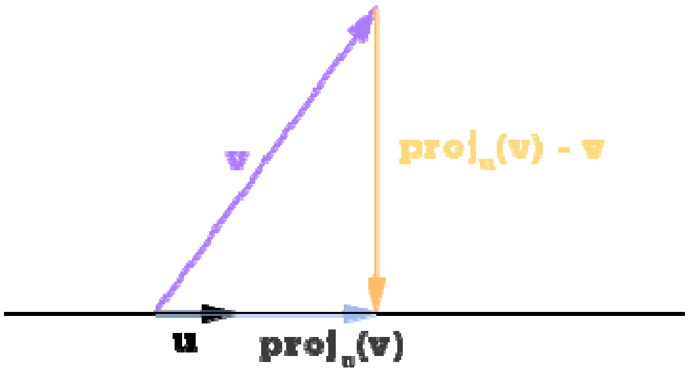
$$\|u\| = \sqrt{< u, u >}$$

We now come to the key ideas at the heart of this project:

Given a vector u in F, the orthogonal projection of vector v in F onto u is defined as

$$P_u(v) = \left(\frac{< v, u >}{\|u\|^2}\right)u$$

We will also say that P_u (v) is the *shadow* cast by v on u, or the component of v in the direction of u.



Given a set U = {u_1, u_2, ..., u_k} of linearly independent vectors, the *decomposition* v_u of vector v with respect to U is the set of shadows cast by v on every vector in U. The *reconstruction* of v with respect to U is the sum of the shadows cast by v on every vector in U. (The reconstruction is a linear combination or weighted sum of the vectors in U whose coefficients or weights depend on the projections of v onto the vectors in U). Note that every reconstruction is an approximation; it is not necessarily true that v = v_u. The larger the size of U, the better the reconstruction.

In this project, the vectors in U will not only be linearly independent but also orthonormal. A nonempty subset S of vector space V is orthonormal if the vectors in S are of unit length and pairwise orthogonal. There is a technique, known as the Gram Schmidt procedure, which takes a set S of linearly independent vectors and outputs a set of orthonormal vectors with the same size (as well as the same span) as S.

2.2. The procedure

This section details the procedure that generates the 3-channel video projection.

Select two movies M_1 and M_2:

M_1 = *Alphaville* (Jean-Luc Godard, 1965)

M_2 = *Witch's Cradle* (Maya Deren, with Marcel Duchamp, 1943).

Choose at random a set G of 72 linearly independent frames from M_2 and then apply the Gram Schmidt procedure to G, thus generating a set of orthonormal vectors ("the database").

For every frame f in M_1, compute the shadow cast by f onto every vector in the database as well as the reconstruction of f with respect to the database.

Display every frame f in M_1 next to its reconstruction in the center image.

Display the shadows cast by f on every vector in the database on the right and left images. The following is a view of the right screen.

The following is a view of the 3-screens.

3. The chosen footage

The 3-channel installation setup shows the original movie, Godard's *Alphaville*, and its reconstruction in the center image. Its shadow projection onto 72 frames is shown on the left and the right screens. The frames are taken from the (never completed) film *Witch's Cradle*, directed by Maya Deren in collaboration with Marcel Duchamp. The two films were selected because their filmmakers used light and shadow as dramatic elements. Moreover, Godard can be seen a response to the rise of cybernetics and information technologies, while Deren and Duchamp were interested in abstract mathematical spaces. This project is also an artistic response to information technologies and mathematical abstraction. The artist aims to show how mathematical concepts, such as the idea of a vector space and an orthogonal decomposition, can be treated as artistic resources for the generation of unprecedented moving images.

A video documentation, as well as detailed setup instructions, can be found in the project's web site:

http://www.concept-script.com/theorem8/index.html#

Acknowledgments

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