The Influence of Nonlinear Dynamics and the Scaling of Multidimensional Parameter Spaces in Instrumental, Vocal and Electronic Composition

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Abstract

The influence of nonlinear phenomena and the scaling of multidimensional phase space will be presented as generating principles for musical composition. As will be shown, two broad applications seem to have a particularly robust potential for musical expression. The first involves the use of non-linear dynamics to structure large-scale formal development, while the second directly effects local sound production and gesture. A short discussion defining nonlinear phenomena will lead to creative applications found within the following compositions: MAMRE, for solo violin; CANTOR'S DUST, for voice and electronics; DIVERGENCE, for voices and electronics; ANAPHORA, for solo voice, and; STRING QUARTET #1.

In this paper, the influence of nonlinear dynamics and the scaling of multidimensional parameter spaces will be presented as generating principles for musical composition. As will be shown, two broad applications seem to have a particularly robust potential for musical expression. The first involves the use of non-linear dynamics to structure large-scale formal development, while the second directly effects local sound production and gesture. These influences will be demonstrated through my compositions: MAMRE, for solo violin;

CANTOR'S DUST, for voice and electronics; DIVERGENCE, for voices and electronics; ANAPHORA, for solo voice, and; STRING QUARTET #1.

Introduction

Composers, performers and listeners of contemporary classical music have long recognized the vitality of complex multiphonic instrumental and vocal sonorities. However, until recently the theoretical understanding of these complex states relied upon the methods of mechanical reproduction (i.e. fingering charts with embouchure indications) while largely avoiding scientific questions [1]. This absence of quantitative data combined with artistic products featuring non-scalable 'extended' performance techniques had the real and unfortunate effect of contributing to the stereotype that complex sonorities were merely tricks incapable of achieving real musical expression. Then, beginning in the early 1980's, theories of nonlinearity were beginning to be applied to complex musical signals that led some to reconceptualize their understanding of the elements involved in the production of sound. In this paper, I wish to argue that one result of this reconceptualization led quite logically to multidimensionality of extra-complex musical sonorities. Most often, this increased awareness has been applied in computer musical contexts. However, as acousticians, programmers, composers and performers had begun to systematically look into the tiny bits of sound, they found that it was possible to pull apart the texture of instrumental and (less often) vocal production. One of the pragmatic ways this was and is done is to look at the elements involved in the production of a sound and to explicitly change certain variables one by one while keeping the others constant. One such composer preceding current directions was Giacinto Scelsi, who often typically composed works that would feature a single tone for lengthy durations, while shifting particular variables within a sound, as in the following excerpt "L'âme ailée".

Currently, my work is involved with multidimensional networks whose internal variables are shifted within an scalable environment. My use of the term scalable suggests that a variable is assigned a minimal and maximal value. Then between these extremes, more or less discrete values are inserted, so that a sequence of linear steps from low to high, slow to fast, etc. is developed. The value of scaling these parameters affects global compositional ratios of novelty versus redundancy. Significantly, this suggests that the logical procedures of composed sound may stretch across 8 to 10 dimensions, rather than the usual 2 (pitch and rhythm). Then in certain cases, as the parameter space is filled with an increased activity of non-idiomatic behavior, bifurcations may appear to push the output into nonlinear phenomena appearing as unexpected, transient or extra-complex musical sonorities. Before continuing with a description of how these phenomena are used in my work, perhaps a short introduction into nonlinear phenomena will be useful.

Nonlinear Phenomena

Nonlinear phenomena have been reported in many diverse disciplines, including physics, health sciences, engineering, literature, neurology, geology and music. Directly relevant to this paper, nonlinear phenomena related to sound production has been reported for newborn cries [2], pathological voices [3], extra-normal 'extended' vocal technique [4], animal vocalizations [5], flute [6], oboe [7], saxophone [8], trombone, crumhorn, bassoon [9], trumpet [10], and violin [11].

Analysis of real-world phenomena using methods from nonlinear dynamics are frequently based on descriptions of a system within a phase space. The phase space is built from dynamical variables that are necessary to determine the state of the nonlinear system. At every moment, the behavior of a system may be represented by a single phase space point. It has been found that phenomena frequently reach a particular dynamic regime after initial transients. This regime corresponds to a geometrical object in phase space and is termed an attractor. Four types of attractors have been identified: 1) Steady state, a behavior whose variables are constant; 2) Limit cycle, periodic behavior (repeating itself continuously); 3) Torus, a two-dimensional object in phase space that results from the superposition of two independent oscillations; 4) Chaotic attractor, a nonperiodic behavior that never repeats but stays within a limited space [12].

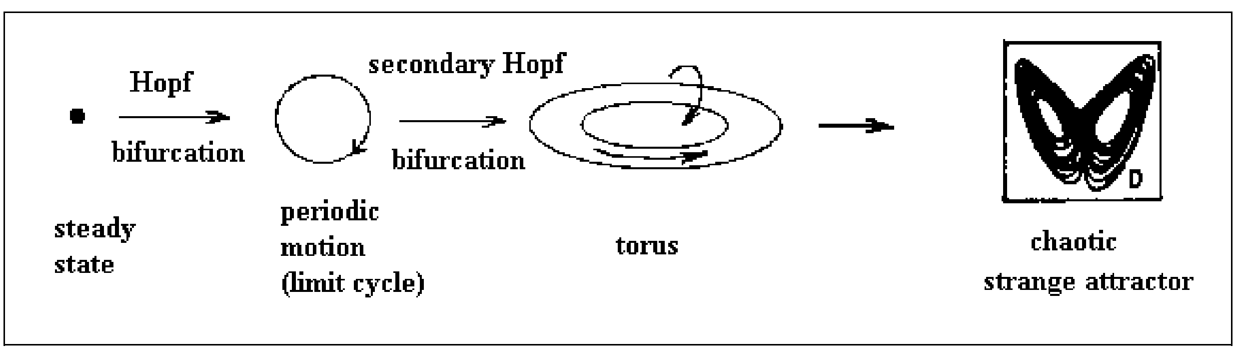

Fig. 1: attractor states with associated bifurcations

Attractors govern the dynamics for constant external parameters such as vocal fold tension or subglottal pressure, in the case of phonation. Often these parameters vary slowly and may feature sudden transitions to new attractors. These transitions are termed bifurcations and include: 1) Hopf bifurcation, a transition from a steady state to a limit cycle; 2) Period doubling bifurcations, transitions from a limit cycle to folded limit cycles; 3) Secondary Hopf bifurcation, a transition from a limit cycle to a torus. Further, subharmonic bifurcations and tori often are precursors of deterministic chaos, such that small parameter shifts induce jumps to nonperiodic oscillations. A comprehensive visualization of transitions can be achieved by bifurcation diagrams [13] which display different dynamical behavior depending on one or two varying system parameters.

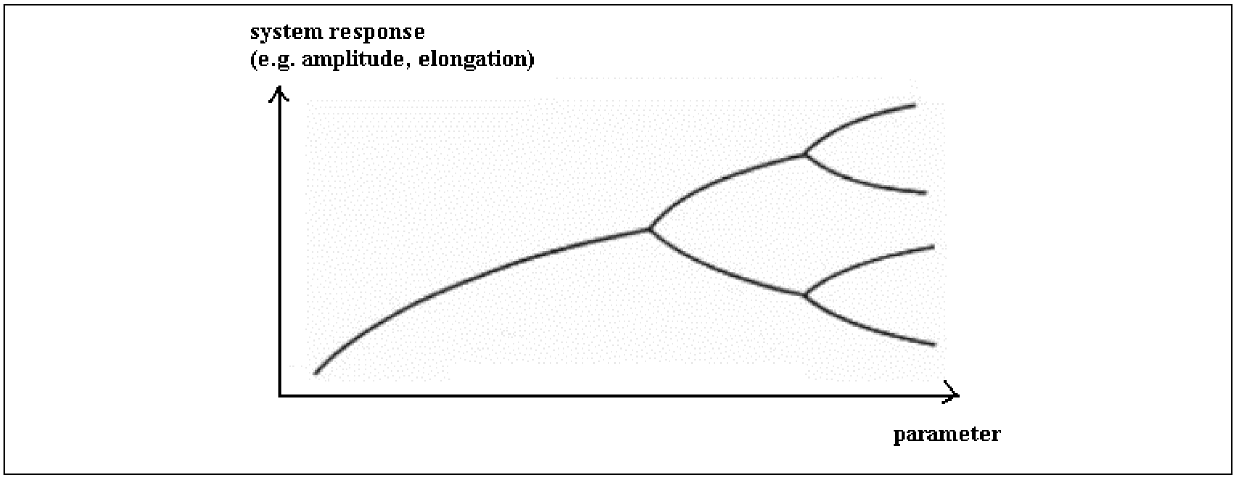

Fig. 2: one dimensional bifurcation diagram

As applied to voice, steady state behavior occurs when the vocal folds are at rest. Then as subglottal air pressure begins to rise a Hopf bifurcation occurs to push the steady state attractor into a limit cycle as the vocal folds begin to produce normal periodic vocal fold vibrations. Often during speech and song, period doubling bifurcations occur and lead to subharmonic oscillation. Subharmonics may be classified as a folded limit cycle, that often appears via transitions from periodic oscillation to an oscillation with alternating amplitudes, or as an addition of a second periodic source, locked at a frequency ratio of 1:2.. Less frequent, though still seen in speech and song are phenomena featuring two or more independent frequencies. This phonation, classified as a torus, may be produced with (left-right) asymmetrical vocal fold vibration and has been termed biphonation [14]. As mentioned above, subharmonics and tori often are precursors of deterministic chaos, which includes high airflow multiphonics as an example.

Applications

Fig. 3 shows bifurcation diagrams showing the experimental results from excised Larynge Experiments. Both diagrams show asymmetries of vocal fold adduction as experimentally applied to excised canine larynges. In Fig. 3a, we see the results that an increase or decrease of micrometer asymmetry (x axis) and subglottal pressure (y axis) produces. With low subglottal pressure, an increase of applied asymmetry had no effect on phonation, as it remained within a chest-like vibration. However, as subglottal pressure increases, irregular vibrations (including period three subharmonics, transient Fo) and periodic single vocal fold oscillations were observed. Likewise in Fig. 3b, an increase or decrease of micrometer asymmetries and subglottal pressure led to chest-like vibrations, falsetto-like vibrations, vortex-induced (whistle-like) vibrations and instabilities. For the larynx shown in 3b, it might be interesting to note that an increase in subglottal pressure at low to medium asymmetries did not result in instabilities, but rather, remained in chest-like vibration – however, as might be expected an increase of asymmetry coupled with an increase of subglottal pressure produced instabilities [15].

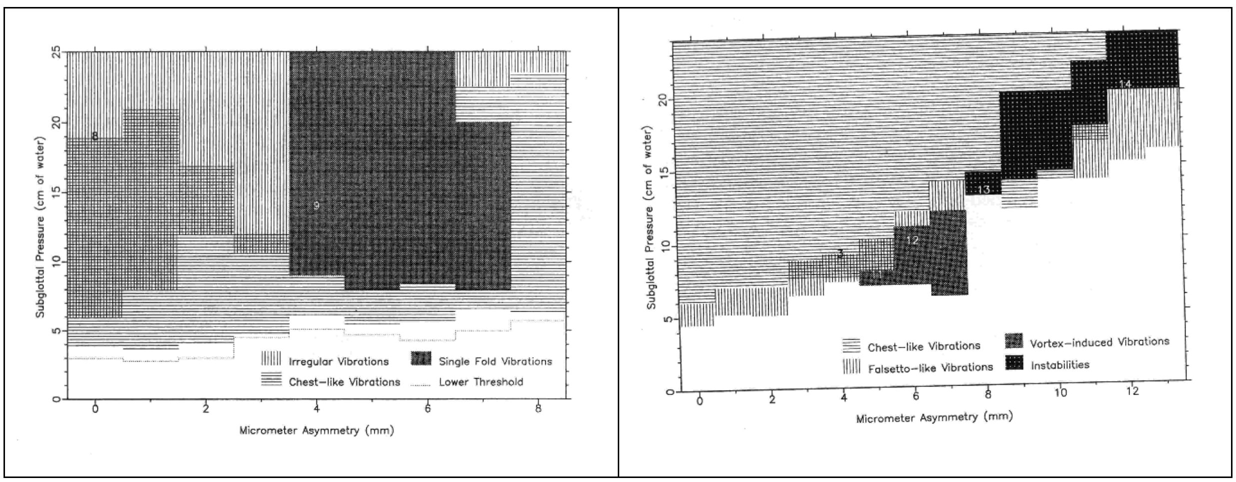

Fig. 3: bifurcation diagrams associated with bifurcations in excised larynge experiments

Closely associated with the well-known butterfly effect, in which small changes of initial conditions may produce large effects in the systems output, these bifurcation diagrams provide experimental evidence that small perturbations may lead to nonlinear results in a musical instrument, the voice. Therefore, the idea of shifting sound production variables in instruments and voices is directly linked to experimental research (as well as to traditional, world music and electroacoustic/computer music experiences). Next, two compositions that utilize the shifting of variables within a multi-dimensional parameter space will be discussed.

Mamre

In MAMRE, a short study for solo violin, a select group of variables were chosen that would offer a closer look into the micro-sound world of the violin. More specifically, the intention was to develop a multi-parameterized network that would allow compositional coherence to be developed across multiple dimensions. Aurally, the result of such a framework resulted in the production of irregular and transient sonorities by shifting inherent variables of sound production into non-idiomatic ratios, when compared with pitch, rhythm and tempo. In addition to rhythm and pitch, the following variables were selected: bow rotation, bow speed, microintervallic movement, microintervallic tuning (Beats), bow placement, bow pressure, decoupling bow speed from tempo, two dimensional vibrati. The results of these manipulations include transient source and spectral segments; complex harmonic and inharmonic multiphonic sonorities; subharmonics, and; inharmonic glissandi. Next, a few spectrographic analyses (with accompanying recordings) may assist in this discussion. The analyses of Mamre were taken from a studio recording by the violinist Chatschatur Kanajan of the Kairos String Quartet [16].

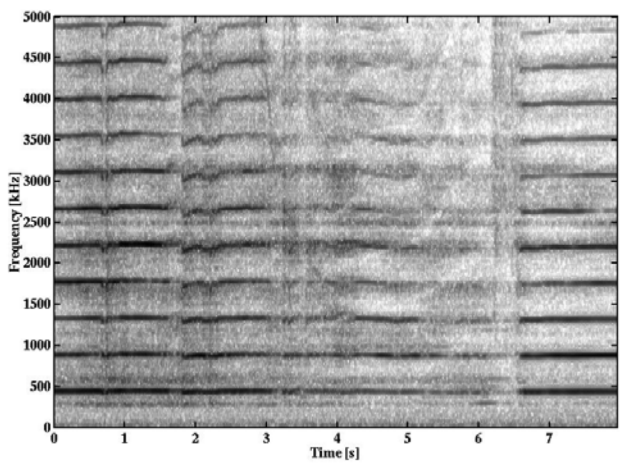

Fig. 4: ord to col legno to ord; ¼ tone glissando; bow placement glissando

In Fig. 4, the results of shifting from ord to col legno to ord, with a ¼ tone glissando, featuring a glissando of bow placement from normal to tasto 4 (hi over fingerboard) to normal are shown. Beginning with a somewhat normal tone, the image shows a disruption near 2" where the bow is shifted to col legno. Then from approximately 3" to 6.5" a filtering of the spectrum occurs to reduce the amplitude of all harmonics (even the fundamental), and above the fourth harmonic significantly more energy is reduced. The result becomes muffled and transient, completely stopping the tone near the 5-6" timing. As well, an inharmonic glissandi appears to be the result of the motion of the wood sliding up and then down the string.

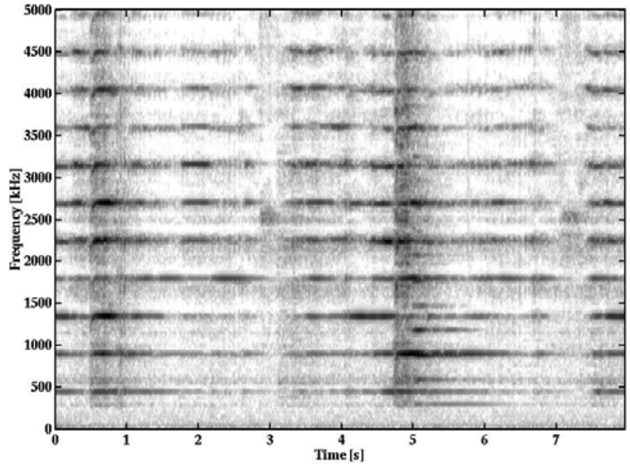

Fig. 5: fast bow speed; change of bow placement

In Fig. 5 an extremely quick bow speed, using approximately ¾ of the bow is combined with changes of bow placement. The prominent effect is of spectral transience. At approximately 5 seconds, an inharmonic band is produced through heavy bow pressure, slightly sounding the d-string.

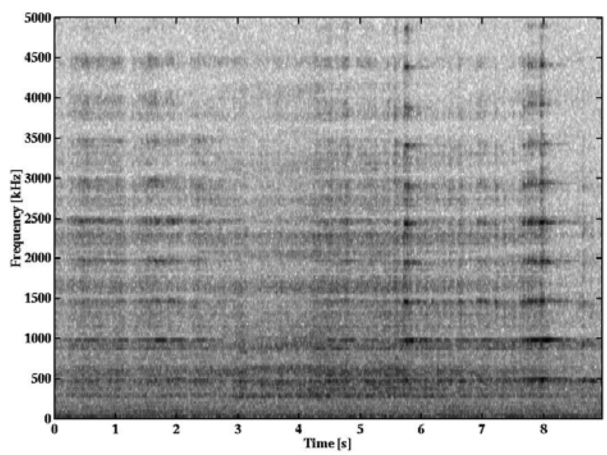

Fig. 6: wood alone; bow placement from ponticello 2 to bridge

Fig. 6 features a sequence in which only wood is used to produce the source sound, while the bow placement switches from ponticello 2 (next to the bridge) to directly on the bridge. The resultant sound is a complex tone of harmonic and inharmonic components. Note how the spectral components shift over time. From 0-2", the harmonic energy is clearly defined, then from 2-4.5" the harmonic energy is significantly reduced. From 4.5-9" the spectrum features bursts of harmonic energy, with loud bursts at 5.8" and 8".

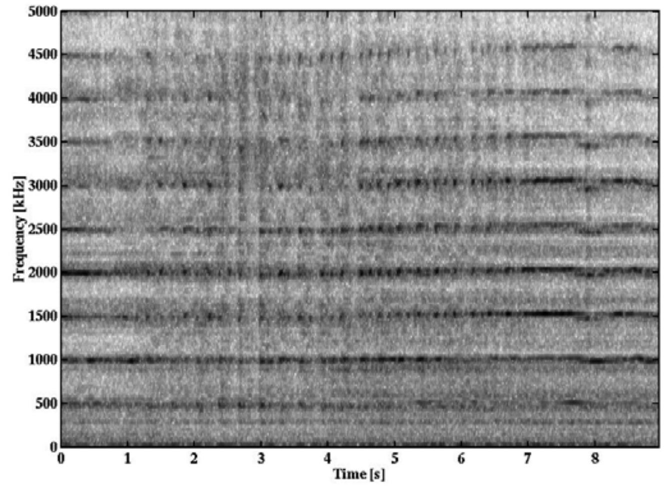

Fig. 7: slow (as slow as possible) bow speed; oscillation of bow placement between ponticello 1 and ponticello 2; high to maximum microtonal pitch detuning between adjacent strings (d, a)

Fig. 7 features a slow bow speed, with an oscillation of bow placement between ponticello 1 and ponticello 2 and high microtonal pitch detuning between the d and a strings. Note the rhythmic disruption to the spectra caused by the sliding hairs of the bow and the extremely slow bow speed, that serve not only as an almost percussive event, but also to filter out particular frequencies. Note as well as the relative strength of harmonic energy from 4.5-8" that results from the placement of the bow so near the bridge.

As the previous examples show, MAMRE, influenced by the performance practice of an al-rabab (a two-stringed Egyptian dichord), begins to explore the micro-sound world through perturbations within the multidimensional parameter space. The results are positive and directly mirror observations of our real, physical and nonlinear world through irregular, transient and non-stable phenomena.

Anaphora

ANAPHORA, the second work that was directly influenced by shifting variables within phase space explores vocal multiphonics that are classified as: voiced to voiced, voiced to unvoiced, unvoiced to unvoiced and three or more simultaneously produced source components. Formally influenced by RICHARD II of Shakespeare, ANAPHORA presents an encyclopedic gathering of transient and complex sonorities. From the over fifty different techniques found within the composition, I will present two nonlinear results – with different production mechanisms, a high-airflow multiphonic, and a glottal whistle. However, as opposed to MAMRE, the development of a list of quantifiable variables, that may be shifted in deterministic ways, will be a bit harder to realize with an instrument featuring no buttons, levers or keys to push, pull or depress – the voice. Therefore, any list will be a bit subjective, but nevertheless may be instructive, if viewed as prominent variables that may be shifted higher or lower according to the chosen dynamic regime.

Air	**Source**	**Resonance/Articulation**	**Heightened potentials**
1. Airflow through glottis 2. Subglottal pressure 3. Torso tension 4. phase within breath cycle (ie. End of breath) 5. Support characteristic 6. Body, physical action 7. Air direction	1. tension of folds 2. pitch range to voice type 3. glottal valving 4. laryngeal height 5. open-close ratio (brassy – ord)	1. coupling, resonator-source 2. front-back tongue placement (bright-dark) 3. nasality 4. placement of sound 5. singers formant	1. intensity of sound

table 1: prominent variables to be shifted during extra-normal vocal behaviors

In Fig. 8 the prominent variables involve the scaling of airflow, laryngeal laxness, placement and intensity of production. In context, the resultant sonority opens the composition and functions as a precursor to a raucous "turkey"-like sound that features an intense fortissimo with a high pitch oscillation (tri-tone), combined with glottal stops. The analyses of Anaphora were taken from a studio recording by Rebekka Uhlig [17].

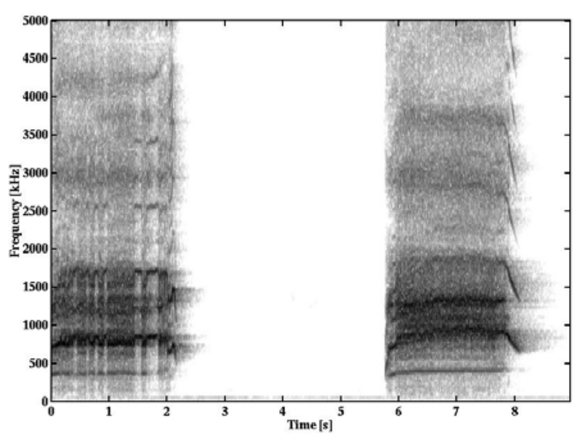

Fig. 8: airflow multiphonic

Note the instances of chaos and spectral transience. In this mutiphonic, the dominant amplitudes appear between approximately 800 and 1600 Hz, as are shown by the intensity of the dark horizontal lines. Compare the first instance with the second instance. The sonic instabilities of the first appear as greater frequency separation between the spectral components, while the second has somewhat of a more uniform band. This is also perceived as the first features somewhat of an oscillatory character, while the second seems more contained or stable. It is important to note that although they both feature heavily inharmonic signals, that they are not uniform along the y axis. In addition, note the band at around 400 Hz - this seems clearly to be a subharmonic at half of the fundamental frequency.

In Figs. 9 to 12, a very special technique that involves low airflow through partially adducted, and presumably, non-oscillating vocal folds is presented. The effect is of a whistle being produced in the glottis (space between the folds). One exciting result of this technique involves the production of a biphonic voice (two or more independent pitches, a torus). Although absolutely rare, it is my experience that this ability to simultaneously produce two or more pitches is available to most people with a functional phonatory apparatus.

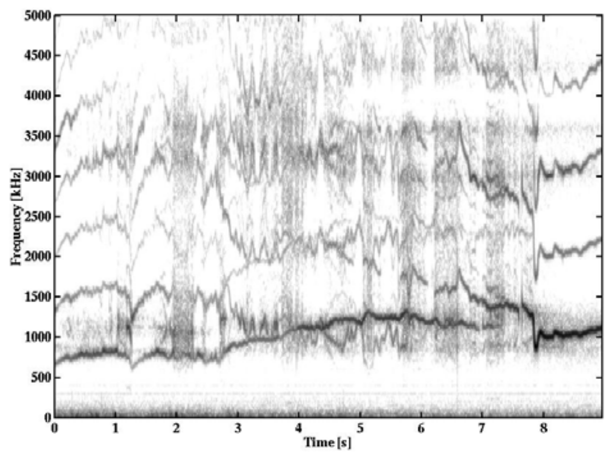

Fig. 9: glottal whistle featuring biphonation

In Fig. 9, note the differing frequency contours that eventually cross from 2.8-8”. This suggests two independent frequencies. Note how both frequencies tend to converge to F1 from 4-6”.

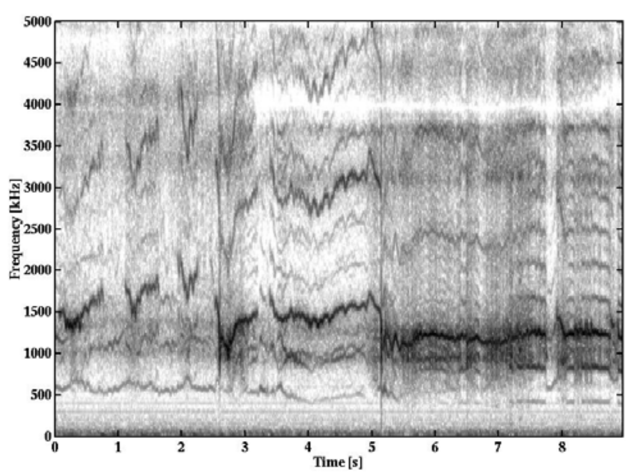

Fig. 10: glottal whistle featuring biphonation

In Fig. 10 from 5-9”, both frequencies lay within the bandwidth of the first formant. Here the frequencies seem most stable, whereas at the beginning the first five segments of the upper frequency seems to be more transient. Also from approximately 550 to 450 Hz a subharmonic of the fundamental frequency occurs and lasts through the entire sequence.

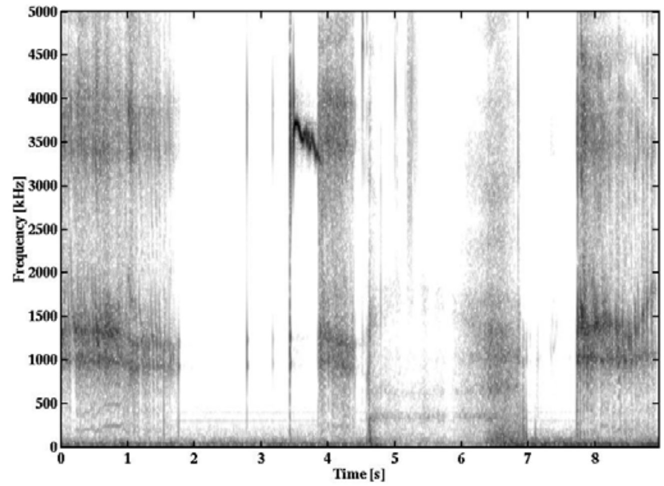

Fig. 11: an extremely high transient

In Fig. 11, remarkably, the highest Fo, I’ve ever seen – that of a glottal whistle around 3.5 kHz near 4”.

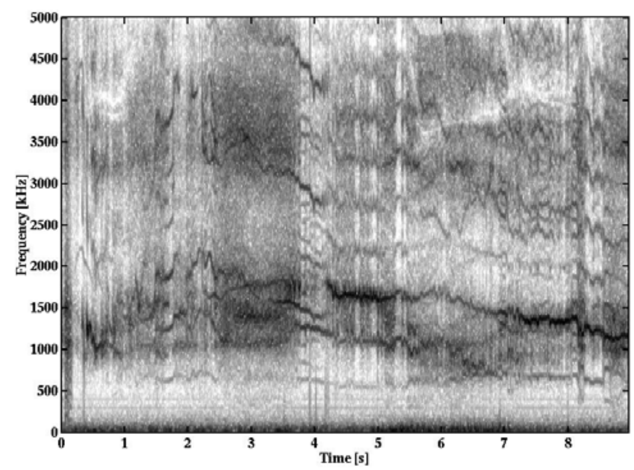

Fig. 12: biphonic glottal whistle

Fig. 12 is significant in a different way, as more than two simultaneously produced frequencies appear. From 2.5-3.5” as many as three or four independent frequencies occur, and around 4” a region featuring two independent frequencies seem to combine with a lower subharmonic frequency.

Like MAMRE, ANAPHORA is influenced by the shifting of variables within a parameter space. Then going further, the sonic, extra-normal results are themselves representative of special attractor classes leading to and involving deterministic chaos. Next, let's take a look at how the influence of nonlinear dynamics has effected global, formal characteristics in the following compositions CANTOR'S DUST AND DIVERGENCE.

Cantor's Dust

CANTOR'S DUST refers to an abstract construction developed by the 19th century mathematician Georg Cantor, termed the Cantor Set. As James Gleick has written, the geometry of this set is quite simple, "begin with a line; remove the middle third; then remove the middle third of the remaining segments and so on. The cantor set is the dust of points that remains. The paradoxical qualities of such constructions disturbed 19th-century mathematicians, but Mandelbrot saw the Cantor Set as a model for the occurrence of errors in an electronic transmission line. Engineers, saw periods of error-free transmission, mixed with periods when errors would come in bursts. Looked at more closely, the bursts, too, contained error-free periods within them. And so on—it was an example of fractal time. At every time scale, from hours to seconds, Mandelbrot discovered that the relationship of errors to clean transmission remained constant. Such dusts, he contended, are indispensable in modeling intermittency[18]".

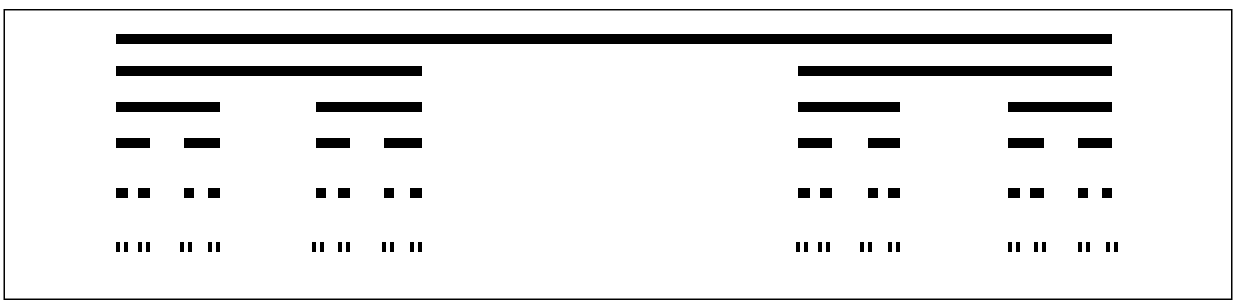

Fig. 13: Cantor Set

CANTOR'S DUST, for voice and electronics, can be performed as a solo work, or as part of a larger stage work titled, CREATION OF THE WORLD. In CANTOR'S DUST, the solo voice is immersed within an environment that models exactly the Cantor Set. For the solo voice, a unique mapping of the vocal tract, developed by the author, becomes the basis for musical exploration. Through this mapping ALL locations within the upper vocal tract are identified, thus considerably exceeding language-based systems of articulation, both in quantity of location and quality of sound output. Compositionally, this framework allows the development of a contrapuntal complex within a single 'face'. This complex is developed through the identification of simultaneously sustained harmonic and inharmonic sound sources, identified as separate strata and produced at various locations within the tract. Add to this heightened vocal fold maneuvers, such as biphonation, glottal whistle (flageolet or whistle registers, vortex-induced vibrations, pfeifstimme, bell or flute registers), complex oscillatory states and we have an exciting environment for many nonlinearities[19].

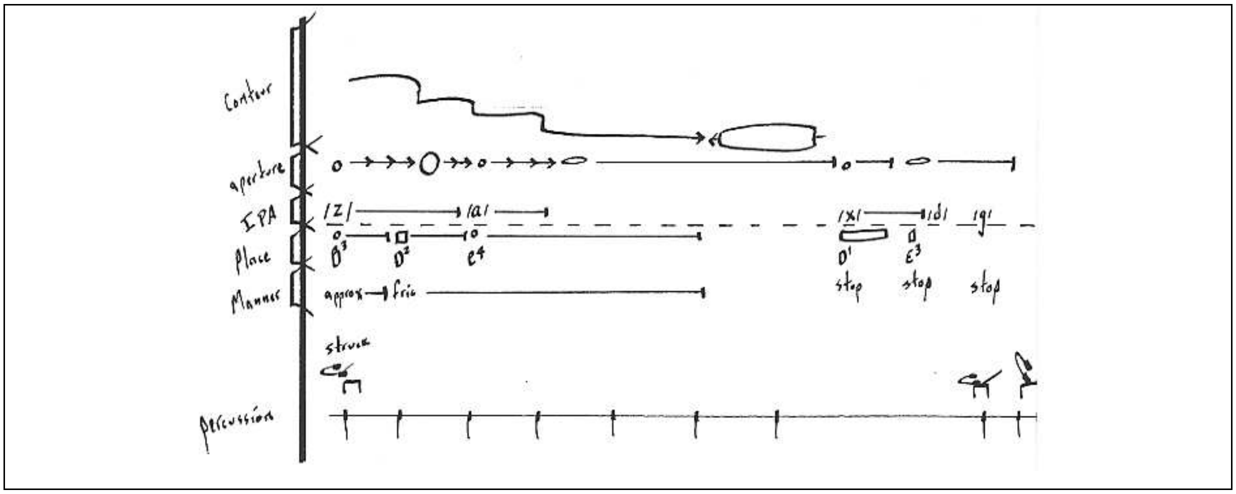

Fig. 14: score of Taffy Twister for solo voice by author [20]

Then, the metaphor of an electronic line featuring long periods of error-free transmission, mixed with periods filled with bursts of dust (errors) perfectly suited the relationship between soloist to accompaniment; palatal mapping to language-based articulation, and thus providing an explicit model of intermittancy. The intended expression was to support the voice through long, sustained tones with slight, transient bursts. The voice is of Rebekka Uhlig. The work was realized at the Elektronisches Studio der TU Berlin.

Divergence

DIVERGENCE refers to a graphic of weather divergence, developed by Lorenz, in which miniscule irregularities in the initial conditions become responsible for potentially large changes. In DIVERGENCE a graphic developed by Lorenz (fig 15) became the model upon which the electronic accompaniment was based. The procedure involved tracing this graphic, within a nonlinear time and pitch shifting audio processing function of the software program Sound Designer, upon another source signal. Due to the nature of the electronic to voice relationship, I wanted an accompaniment that would function similar to CANTOR'S DUST. Therefore, I decided to work with a single source material of extremely long duration. The source consisted of another work of mine titled THE ELEMENTS OF RISK IN CREATION – A SUBATOMISTS VIEW. After assigning two monophonic copies, each to a separate channel, I reversed the time series of one (retrograde), while applying retrograde and inversion functions to the other. Then, to both of these signals I applied the nonlinear pitch and time shift function by tracing onto one channel the blue line, while on the other channel, the red line. Above center position, pitch rose and time shortened, while below center position, time lengthened and pitch lowered. True to the Lorenz graphic, the source material, now separately processed according to different frequency and temporal characteristics, begin similarly, but shift dramatically near the midway point. Over this accompaniment, the solo voices perform specific techniques designed to heighten special nonlinear results. In total, the techniques involved glottal whistle; pitch as low as possible, the go lower; singing at the end of the breath; scrunch; breath sounds. The voices are of Rebekka Uhlig and Jürgen Neubauer. The work was realized at the Elektronisches Studio der TU Berlin [21].

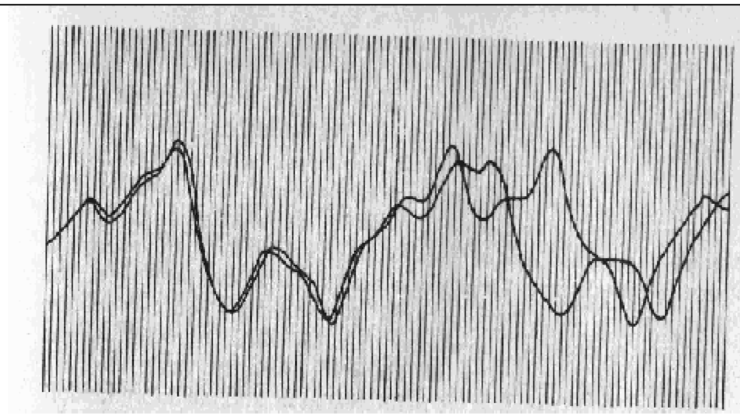

Fig. 15: Lorenz graphic of weather divergence

String Quartet #1

Last, my STRING QUARTET #1 (in progress) features my fullest treatment of the global and local applications of nonlinear dynamics in my work. Although not yet performed, it is felt that from the previous examples, the central concepts will be clear.

Fig. 16 shows an excerpt from movement 1 [22]. On this page are found all of the variables notated within a scalable framework. These variables include: bow rotation; bow angle; bow portion; bow length; effleure (left hand pressure); bow speed; pitch; rhythm; intensity; placement; bow attack and release.

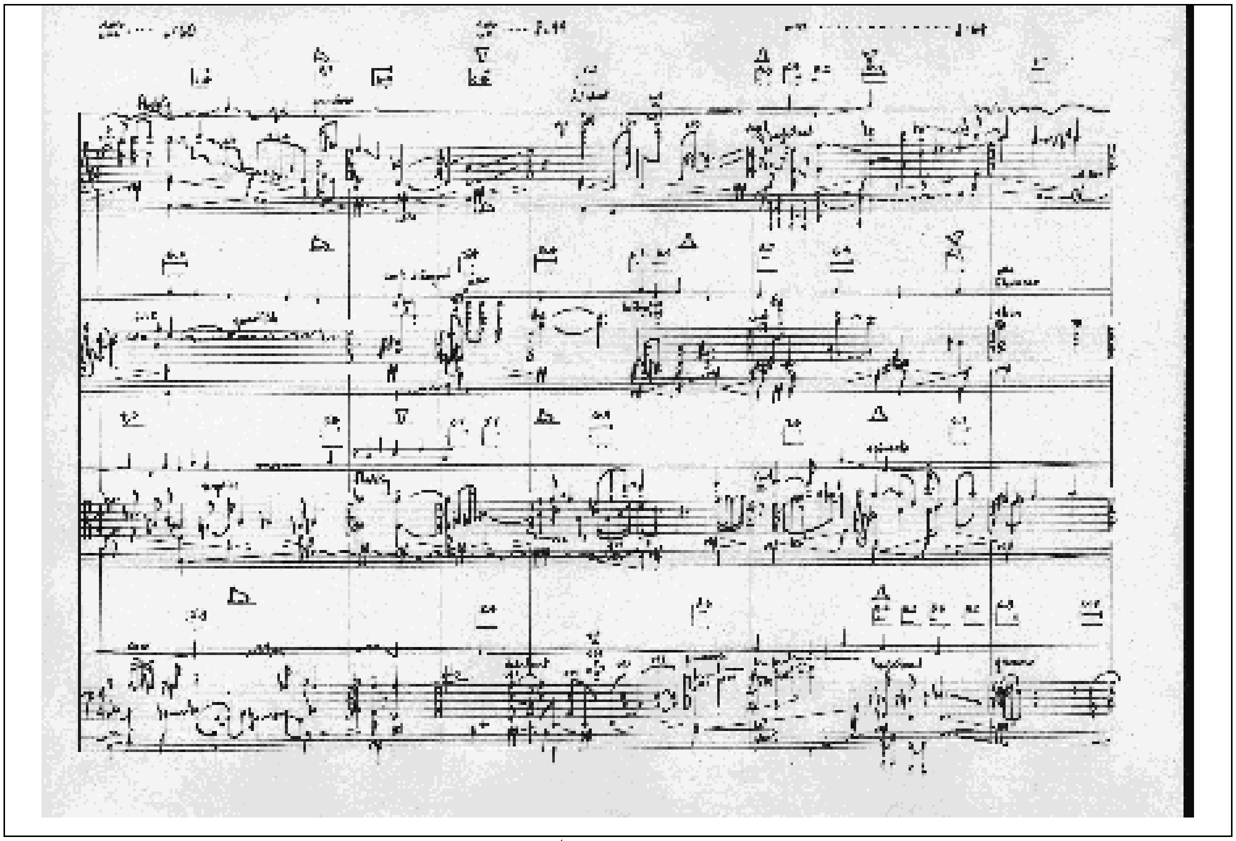

Fig. 16: excerpt from String Quartet #1, 1st movement, by the author

As was seen in MAMRE, the result of shifting variables mostly one at a time produced extra-complex musical structures featuring a crucial property of transition and change. Here, it might be readily apparent that simultaneously shifting multiple variables feature increased bifurcations that impact the resultant sound in often deterministically nonlinear ways.

Formally, movement one references sonata form (without its tonal implications) and pretonal contrapuntal complexes. Nonlinear dynamics becomes part of the functional development through tempi relations that are formed from the fractal dimension found within the outline of the Koch curve (in which an infinite length fits within a finite area, such as may be found in a coastal outline), at an increase of 1.2618. Meanwhile, the treatment of multi-dimensional frameworks is scalable and complete.

Movement two is formally a musical riddle, influenced by the composition UT, RE, MI, FA, SOL, LA by the renaissance composer John Bull, in which the procedure of transcribing letters to integers serves as a formal, generative process [23]. As well, the style of this movement references a typical slow second movement, but significantly diverges to other regimes as the multi-parameterization assumes a heightened characteristic.

Movement three presents an alternative view of the concept of divergence. This time, not through slight differences in initial conditions, but rather through the decoupling of bow speed and portion when compared with tempi. In practice, this is a highly nontrivial situation, as string players are taught to correspond bow speed with tempi. Even slight deviations from this norm often result in non-musical performance and in this string quartet present significant psycho-physiological hurdles to navigate around. In order to explicitly heighten this decoupling, easily identifiable gestures were chosen, that are to be performed at extremely quick tempi. The resultant effect should be dramatic, and if truly dedicated, will features unstable, transient episodes.

Movement Four is based upon the unfolding within the phase space of a strange attractor (a behavior that is stable and non-periodic, that stays within a definable phase space, yet never quite repeating). For the fourth movement, the image of a spiral confined by a box, infinitely deep and not quite repeating became the organizational principle. The process involved abstracting a few geometrical shapes that were subsequently unfolded, so as to fit within a two-dimensional graph, identified as pitch versus time. The effect is one of sliding glissandi, which are then radically shifted according to the results of multidimensional scaling.

Movement five is the most complete treatment in the application of nonlinear dynamical thinking to form and production. In this movement I translate the logistic equation (a model of long-term population change) onto the multidimensional parameter space (see figure 2). When the control parameter is less than 1, all iterated values decay to zero (meaning that a particular population becomes extinct), and not much use for my musical intentions. Then as the control parameter increases between 1 and 3, iterations converge to a single value, and still not much use for my purposes. Between 3.0 and approximately 3.58, a series of bifurcations occur that are highly sensitive to the value of the control parameter. First it converges to two final values, then continuing to 3.5, four values result, and continue to increase until approximately 3,58, in which chaos appears. Between 3.58 and 3.99, the behavior is not purely chaotic, as windows of periodicity occur within this information-rich field [24]. As the period-doubling, -tripling, and -quadrupling is musically limited, I decided to focus on the deeper levels within the cascading series at values above 3.57. This governs localized form. Globally, the fifth movement is governed by the Mandelbrot set, in which large active regions are joined by thin, filament-like strands to far-reaching islands. The metaphor provided by the long filament-like strands provided the right cues for developing purely inharmonic and low-amplitude sections.

To summarize, nonlinear dynamics has been important in my work both globally (formally) and locally (sound production/gesture). Closely linked, concepts of multi-dimensionality and scalability influence technical developments that shift learned methods of performance into networks of non-idiomatic ratios, that result in extra-complex, transient and possibly non-linear phenomena - in short, offering powerful methods and rationale for continued technical and expressive exploration.

To end with a quote from James Gleick, “Equally critical in biological systems is flexibility: how well can a system function over a range of frequencies. A locking-in to a single mode can be enslavement, preventing a system from adapting to change. Organisms must respond to circumstances that vary rapidly and unpredictably; no heartbeat or respiratory rhythm can be locked into the strict periodicities of the simplest physical models ... Goldberger noted “fractal processes associated with scaled, broadband spectra are ‘information-rich’. Periodic states, in contrast, reflect narrow-band spectra and are defined by monotonous, repetitive sequences, depleted of information content.” Treating such disorders, he and other physiologists suggested, may depend on broadening a system’s spectral reserve, its ability to range over many different frequencies without falling into a locked periodic channel.” Then quoting Mandell, “is it possible that mathematical pathology, is chaos, is health? And that mathematical health, which is the predictability and differentiability of this kind of structure, is disease [25]?”

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