Organic visualisation of high-dimensional objects: exploration of a world of forms based on the heart transform

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Abstract

The paper starts with a brief motivation for the present type of work. The ecological argument, the world view argument and the argument in terms of understanding intuition are summarized. Then, a particular approach to the generative construction of organic forms is developed. The forms are based on variations of the 'heart transform'. These variations allow one to construct two- and three dimensional visualizations of high-dimensional objects. The resulting forms can be described as 'organic' and complex, but some basic features, such as the requirement that a construction should be broader at its basis than at its top, or that it should be more fractal at its boundary than at its center, can be controlled. The representations of some sets of high-dimensional points are aesthetically more attractive than the representations of other sets. It is discussed how this leads to the possibility of an aesthetic - instead of an algorithmic- basis for decisions on class-membership, and how this may shed light on the difference between algorithms and intuition.

1. Introduction

In this introductory section, we spell out our three main reasons for doing the present type of research. We differentiate between the cognitive-ecological argument, the world view argument, and the argument concerning the understanding of intuition. A new approach to the construction of generative organic forms and its context are described in the next sections.

1.1 The cognitive-ecological argument

It has been conjectured that different types of visual environments are processed in different ways by brains [1]. Environments with relatively simple part-whole structure can be represented by 3D-representations. Such representations have hierarchical structure. At each level, an object, or part of it, is described in terms of simple forms or 'geons' [2] (see also [3]). At the next level, a geon can be split in different geons that correspond with a more detailed representation of part of an object or scene, and so on. 3D-representations have been called 'schematic' or 'topological' images [3]. The word 'schematic' refers to the fact that they are much more simple than the perceptual patterns from which they are derived. Many kinds of variations are filtered out during the process that leads to their identification. They are associated typically with concepts at the basic level of abstraction [4]. Due to the prominence of these concepts in verbal knowledge, these images contribute in an important way to the meaning of words. The term 'topological' refers to the fact that the relations between parts of such images are not specified in precise, metric terms, but only in approximate, topological terms.

3D-representations are crucial for the representation of functional information, as well as for reasoning about functional relations. This is due to the fact that the part-whole structure of an object is very informative for the types of function that an object can have [5]. Further, the relative remoteness from perception of this type of image allows for manipulations in the images that are not direct reflections of changes in the perceptual environment. They are suited for contemplation of variations in part-whole structure and they may lead to the detection of new functional properties.

Still, these images have a number of limitations [4]. In different contexts, they are too remote from perception in order to still allow concise recognition of stimuli in the outer world. For instance, a schematic image of a face allows one to identify the face as a face, but for subtle variations of non-verbal facial communication, the representation is not rich enough. Also, such images are too imprecise to be of use as a basis of spatial locomotion. Further, they are not suited as a basis for aesthetic appreciation. Such processes need another type of image as a point of departure.

It can be conjectured that the latter do not result from 3D-representations by addition of significantly more geons, or by extending them with other types of information (such as information about texture or color). The dependence of 3D-images on topological relations makes them too imprecise for particular recognition tasks and for aesthetic appreciation of complex environmental stimuli. Further, a brain appears to have upper bounds on its structural capacity when hierarchical structures are concerned. In linguistic contexts, sentences with too many nested sub-sentences become non-understandable. The tight relation of 3D-representations with linguistic processing suggests that also here, the structural complexity of representations is subject to upper bounds. Therefore, another, less abstract type of mental image is required. These 'metric' images have been assumed to be more close to the 2-1/2D stage in Marr's schema [4].

For our present concern, it is of importance to notice that some external stimuli are more prone than others to give rise to processing with help of 3D-representations. Typically, man made objects that are inserted in the landscape are put there with a particular function. The part-whole compositions of the objects are dictated by this function, and a 3D-representation reflecting this composition is straightforwardly generated. This contrasts with many types of natural environments. A mountain landscape, a place in a wood, or even a single tall tree, is too complex in order for a concise 3D-representation to be generated if the latter is of bounded complexity. This means that human intervention in the landscape gives raise to an increase in stimuli that can be processed in one particular way. As a consequence, people more often activate this type of representation, and the necessity to exercise other types of representation fades.

If less occasions leading to the activation of one type of representation are present, the proximal cognitive processes (i.e. the processes taking this type of representation as input) are triggered less often too, and may even decrease in subtlety, as the self-organizing brain depends on training to refine the differentiations it makes. This way, a vicious circle appears: people change their visual environment by replacing natural forms with 3D-codable ones. As a consequence, their brains become less specialized to process natural environments, and the appreciation of the latter becomes based on coarser cognitive processes. And hence, the pressure to further replace the natural environment can increase, for one cares less if things that are not fully appreciated disappear. Then, opportunities to exercise processing of natural environments further decrease, and the circle is closed. Hence, organic architecture may play an important cognitive-ecological role. If it is processed in the way in which natural environments are processed, neural processes that are specialized in natural environments can be activated, and the vicious circle can be counteracted.

This line of thought was anticipated by philosophers like Heidegger [6], who differentiate between functional environments (Bestand) and non-functional ones (Gegenstand). When the former type of environment starts prevailing in a systematic way, our daily consciousness is affected much stronger than may appear at first sight.

1.2 The world view argument

The argument of section 1.1 applies to any kind of architecture that uses organic forms, generative or not. The arguments of 1.2 and 1.3 only hold for particular generative methods. In renaissance times, architecture reflected a view on the world, and a particular way of dealing with knowledge. Scientific knowledge and intuition were expressed in major architectural works. Once created, these works in turn were used as frames for further structuring knowledge (see the classical work of Yates [7]). Through the ages, the link between art and science has remained, not only in the sense that science and technology allowed for new material substrates or carriers of art, but also in the sense that scientific insights have inspired artists. For instance, about one century ago, around (and even before) the advent of restricted relatively, the art community was fascinated by the idea of higher dimensional spaces, an idea that inspired writers, but also painters like Picasso [8]. These days, genetic art and chaos- or fractal-based art testify of a similar interaction.

In present day architecture, such links are relatively rare. There are, however, a number of possibilities to link architectural constructions with insights into the workings of nature and of the brain:

i. The first possibility that comes to mind is the link between architecture and evolutionary theory (and that has been explored by Soddu [9]).

ii. The second possibility is a link between architecture and fundamental physical theories. Different roads may be followed. For instance, the visual beauty of some of the representations studied in the context of chaos and fractal theory is well known. However, they appear not to have found their way to architectural design. As another instance, it is well known that different fundamental physical theories deal with more than three dimensions. There are mathematical tools to visualize high-dimensional objects. Some of them lead to objects of remarkable organic shape. Section 3 of this paper gives an instance of such an approach.

iii. Generative art has been linked with problem solving. For instance, benchmark problems of neural networks can be solved with help of cellular systems in such a way that the solution is a beautiful fractal [10,11]. The efficiency of these systems (in comparison with other artificial intelligence methods) has recently been demonstrated [12]. Section 4 explains another perspective on the link between art and cognitive problems.

In sum, architecture can integrate different fundamental features of our world view. As such, it can counter the post-modernist scattering of our ‘life-world’, and give more meaning to objects that often have a functional meaning only.

1.3 Increasing our understanding of how intuition may work

The difference between intuitive knowledge and algorithmic knowledge remains a controversial issue in cognitive science and in artificial intelligence theory. Some would say that both are generated by neural networks, whereas others -such as Penrose- postulate a fundamental distinction, and put forward that intuition is fundamentally non-computational [13]. This paper explores the possibility that intuition, in contradistinction with algorithmic problem solving, has a fundamental aesthetic component. Suppose that, in a problem solving context, examples are collected, and that they are grouped into classes. Suppose that examples are represented as points in some high-dimensional space. The method to be explained in section 4 gives a two- or three-dimensional visual representation for both the examples and the classes. Then, purely aesthetic criteria referring to the visualizations may serve as a basis to decide if newly encountered examples can be included in one of the classes, or as guides to define the classes themselves.

Since the method at issue is basically the same as the one that was used to construct forms aimed to be of use in generative design contexts, the philosophical and scientific study of intuition can be straightforwardly linked with these contexts. We wish to see this as an argument in itself, but this point evidently strengthens the argument of 1.2.

2. The heart transform and two-dimensional representations of high-dimensional geometric objects

The approach to organic forms that is pursued in this paper has a particular transformation at its core. It is named the ‘heart transform’ *H* because of the fact that, for certain values of its parameters, it takes the form of a heart. Mathematically, it is a function that maps an n-dimensional space on an n-dimensional space. By application of an iteration process, its ‘amplified’ form *AH* maps an n-dimensional space on an m-dimensional space, where m can be equal to or smaller than n. For instance, *AH* can be used to give 2- and 3-dimensional representations of high-dimensional geometric objects. Suppose that $p=(p_1, p_2, \ldots,p_n)$ is a fixed point of an n-dimensional space I. The coordinates of this point are parameters of *H*. *H* transforms a vector $x=(x_1,\dots,x_n)$ into a vector $y=(y_1,\dots,y_n)$ in accordance with:

$$\begin{cases} y_1 = p_1+(a/b)^q.(x_1-p_1) \\ \dots \\ y_n = p_n+(a/b)^q.(x_n-p_n) \end{cases}$$

with:

$a = \max\{|x_1-p_1|,\dots,|x_n-p_n|\}$

$b = ((x_1-p_1)^2+\dots+(x_n-p_n)^2)^{(1/2)}$

q is a parameter that is put to 2 in the illustrations of this paper (except in case of Figure 9)

H can be applied on all points of I or on a subset S of I. Suppose that n=2, that S is the surface contained in the circle with centre (300,300) and radius 300, and that $p_1=p_2=600$. Then, the image of S by application of *H* is shown in Figure 1. For $p_1=p_2=300$, the image of S is given in Figure 2.

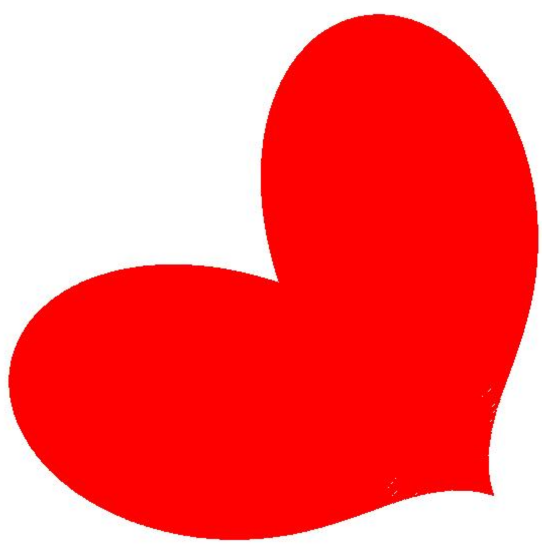

Figure 1.Transform of a circle for $p_1=p_2=600$

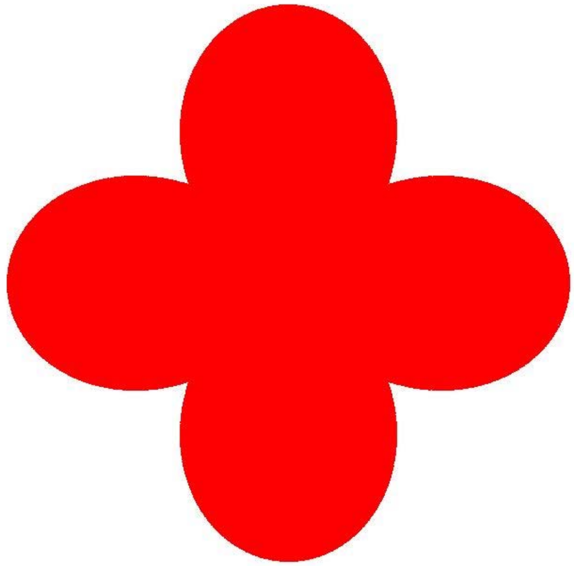

Figure 2. Transform of a circle for $p_1=p_2=300$

Consider a two-dimensional heart transform. Since its output is a two-dimensional point, the transform can be iterated n times. A two-dimensional form can be associated with an n-dimensional vector $v=(v_1,\ldots v_n)$ if, at the k-th step of the iteration, v_k is inserted as a parameter that modifies the transform. Consider a point x that belongs to a subset S of the two-dimensional plane. *AH*(x) is obtained in n iterated steps, where each step has three parts. Suppose that after the k-1-th step, x_{k-1} was obtained. Then,

1. $H(x_{k-1})$ is computed

2. Subsequently, this point is rotated around (p_1,p_2) with a angle $\eta=v_k\,(\alpha+\beta.h.f)$, with:

   α is a constant phase factor and β is a constant

   $h=(b/\gamma)^2$, where b is the distance between $H(x_{k-1})$ and p, and γ is another system constant

   $f=1+\cos(2.k.\sigma)$, where σ is the angular coordinate of x, and k is the number referring to the iteration (so that for the first step, k=1).

3. Finally, the resulting point is subject to a scaling transform relative to p. The scale s is determined by $s=1+\delta.\cos(2.k.\sigma)$, where δ is another system constant

Intuitively, v_k determines the direction and the magnitude of the rotation at step k. The magnitude of the rotation is larger for points that are far from p (this is the meaning of the quantity *h*). This helps to fractalize forms near their boundaries, whereas their interior remains relatively homogenous. The quantity *f*, as well as the scaling operation in step 3, may remind one of the Mandelbrot transform. Also there, the angular coordinate of a point determines its extent of rotation, after which a scaling relative to the origin is applied. But there are two important differences. First, the angular coordinate involved remains the coordinate of the starting point x (and hence is not the angular coordinate of the iterated point $H(x_{k-1})$). Second, unlike in case of the Mandelbrot transform (where the scaling is a function of the distance of a point from the origin), the scaling operation itself depends on the angular coordinate. The former difference prevents volatilities of the form from becoming very wild. The second difference compensates for this reduction in complexity by insertion of a more modest source of fractality. Nevertheless, some of the forms obtained by the present method have some coarse visual Mandelbrot-like features.

We illustrate the procedure for forms associated with eight-dimensional binary points. Figure 3 shows the form that is associated with u=(-1,-1,-1,-1,-1,-1,-1,-1), and that results when *AH*(x) is applied to the inner area of the circle S with center (300,300) and radius 300. For the same set S, Figure 4 shows the form that is associated with v=(1,1,1,1,1,1,1,-1). Both forms are drawn for p_1=p_2=300. The color in the forms is dictated by the distance between x and p. Forms are drawn from outward to inward (so that the images of points closer to p are drawn on top of the images of more distant points).

Figure 3. Form that is associated with u=(-1,-1,-1,-1,-1,-1,-1,-1)

The present method associates a form with every point of a space of arbitrary dimension. The higher this dimension, the more fractal the nature of the form. If high-dimensional, non point-like geometric objects are conceived as sets of points, then they can be mapped on sets of two-dimensional forms. These forms can be combined in different ways in order to obtain a two-dimensional representation of the object. Section 4 has some illustrations of one instance of such a combination.

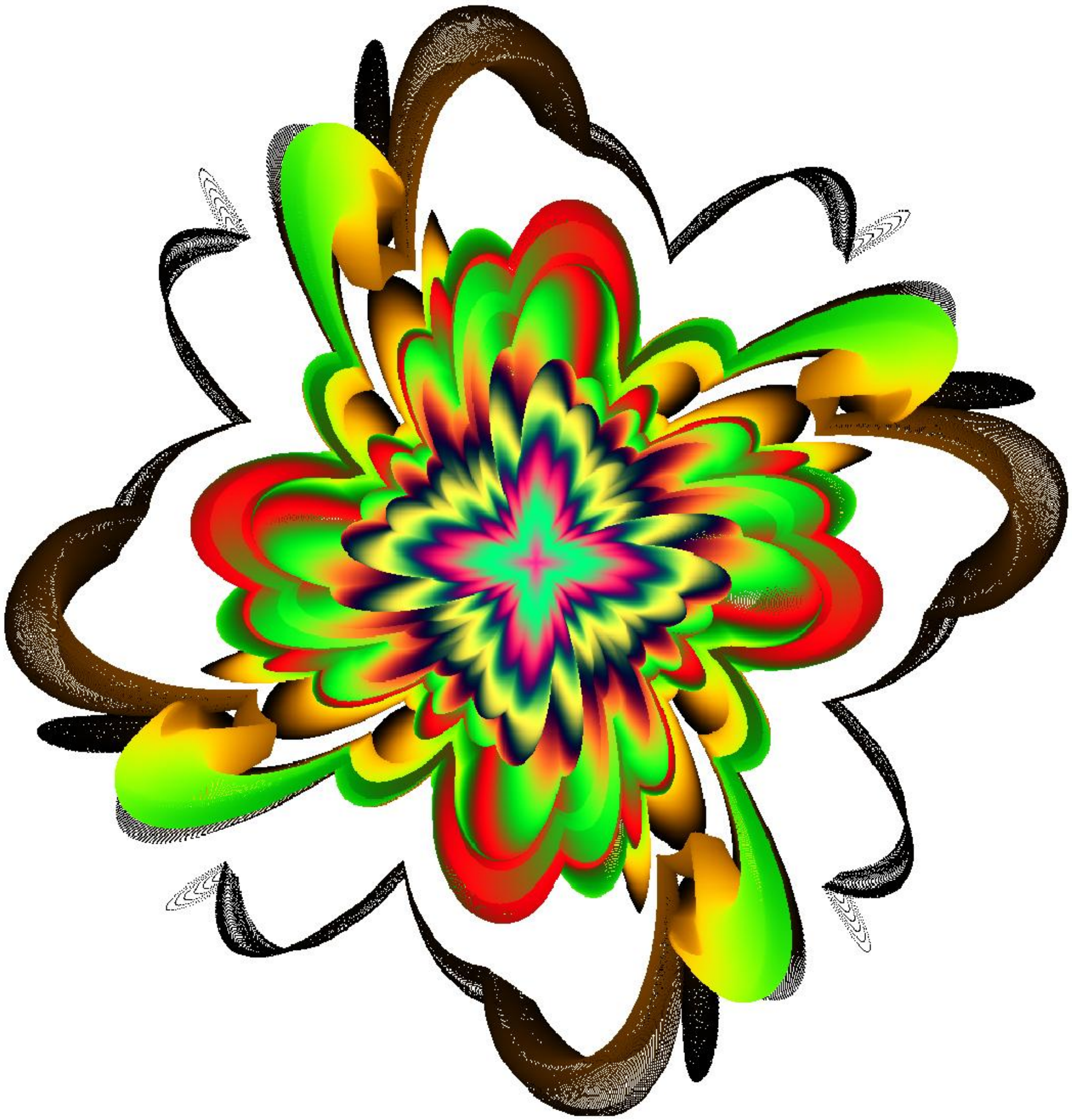

Figure 4. Form that is associated with v=(1,1,1,1,1,1,1,-1)

3. The heart transform and three-dimensional representations of n-dimensional points

3.1 Three dimensional renderings of 2-dimensional images of n-dimensional points

The two-dimensional forms of section 2 can be interpreted as top-views of 3-dimensional objects. Suppose that, like in section 2, b is the distance between a point $x \in S$ and p. Suppose that S is a circle with center p radius r. One can define a cone in three dimensions by the specification that, for $x=(x_1,x_2)$, a third coordinate is defined by $x_3=b$. The height of the cone is zero at the boundary of S, and is equal to r at its center. Every point on this cone can be mapped on a new three-dimensional point by *AH*(x) by the specification that the first two coordinates of the new point are the two coordinates provided by *AH*(x) and that the third coordinate remains equal to b. This way, the original cone is deformed into an organic volume that keeps the global property of being broad at the basis and small at the top. Since in Figures 3 and 4 images of points closer to p were put on top of images of points with larger distance from p, Figures 3 and 4 can be interpreted as top-views of thus deformed cones. Figure 5 shows a side view of the three-dimensional form corresponding to Figure 3. One can use the present method to deform other forms as well. Suppose that half of a sphere is defined on S by specifying a height coordinate for every point x in accordance with $x_3 = (r^2 - b^2)^{(1/2)}$. Then, Figure 6 shows the organic form resulting from the deformation of the upper half of the sphere with *AH*(x) and that corresponds to Figure 3.

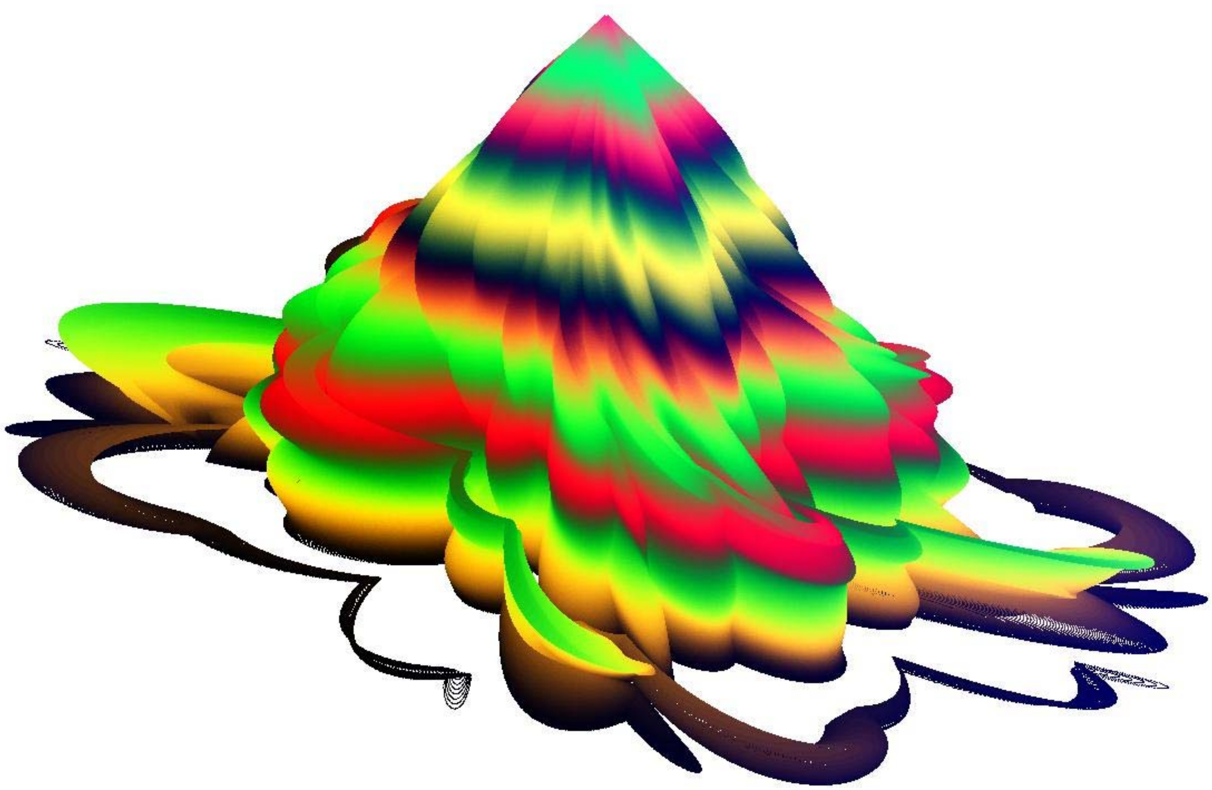

Figure 5. Side view of the cone–based organic form corresponding to Figure 3.

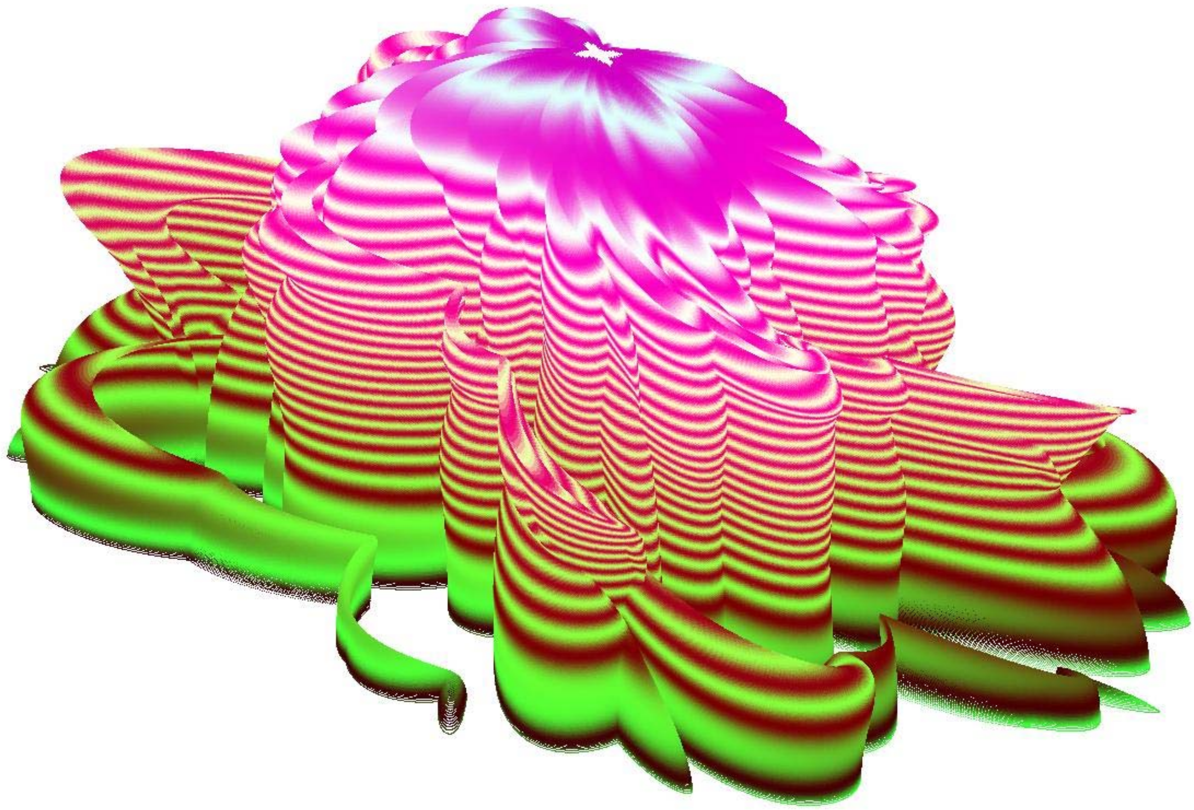

Figure 6. Side view of the sphere–based organic form corresponding to Figure 3.

3.2 Direct three dimensional renderings of n-dimensional points

The previous subsection took as its point of departure the AH(x) function based on the two-dimensional heart transform H(x). It is possible to work more directly in three dimensions by taking the three-dimensional transform H(x) as a starting point. In three dimensions, H(x) operates on a subset S of a three-dimensional space, and it maps S on another subset of the same space. The subset S itself does not have to be three-dimensional. Because of constraints in terms of computation time, we work with two-dimensional boundaries of three-dimensional objects instead of with the entire objects. As an instance, consider a spherical surface with center (300, 300, 300), and with radius 300. Suppose that p coincides with the center of the sphere. Then, the transform of the surface is shown in Figure 7.

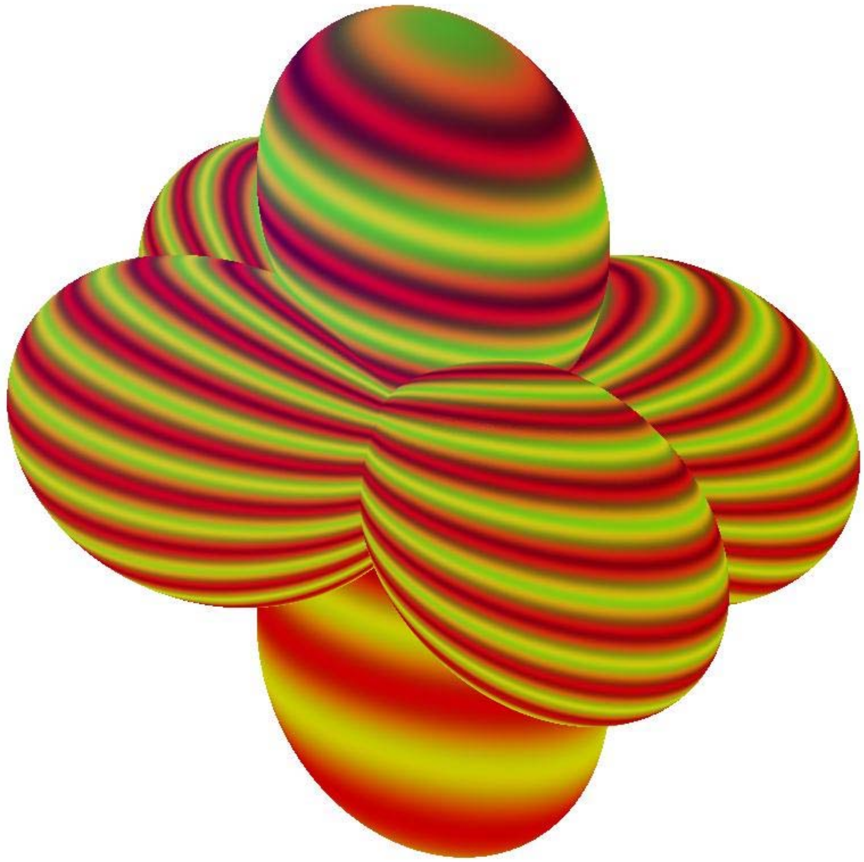

Figure 7. Three dimensional heart transform operating on a sphere and with p=(300,300,300)

There are different ways in which three-dimensional generalizations of AH(x) can be defined. We differentiate between three possibilities:

a. According to the first possibility, the surface in three dimensions obtained by a three-dimensional H(x) is cut in horizontal slices, and each slice is transformed by the two-dimensional transform AH(x). This method is a variation of the one that resulted in Figures 5 and 6; instead of taking a cone or a sphere as a point of departure, the method starts with a form that resulted from an application of a three-dimensional H(x). For this procedure, Figure 7 leads to the form that is shown in Figure 8.

b. The second possibility is an intermediate one. H(x) is used to define a three-dimensional form, then it is cut in horizontal slices, but AH(x) is modified so that some of its parameters refer to three-dimensional properties. For instance, the distance in the numerator of the

Figures 8. Variation of three dimensional AH(x) for u=(-1,-1,-1,-1,-1,-1,-1,-1)

parameter *h* can be made to refer to the three-dimensional distance of a point to p instead of to the two-dimensional distance of a point to the projection of p on the horizontal slice. Figure 9 shows an instance of a resulting form. For the sake of illustration, we put q=1.5 (the value of this parameter is 2 in all other Figures).

c. Third, three-dimensionality can be exploited more fully to obtain a much larger family of forms. For every vector $v=(v_1,\ldots v_n)$, a fully three-dimensional AH(x) can be defined as the result of n three-step iterations. Suppose that after the k-1-th step, x_{k-1} was obtained. Then,

1. $H(x_{k-1})$ is computed

2. Subsequently, the resulting point is subject to a three-dimensional affine transform with v_k as a parameter. One instance is as follows. First, a coordinate plane going through x_{k-1} is selected, and x_{k-1} is transformed in x'_{k-1} in this plane by a two-dimensional rotation (with rotation center the projection of p on this plane and with an angle $\eta=v_k\ (\alpha+\beta.h.f)$) in accordance with the algorithm of section 2. Then, another coordinate plane going through x'_{k-1} is chosen, and again x'_{k-1} is subject to a rotation around the projection of p on this plane and with an angle $\eta=v_k\ (\alpha+\beta.h.f)$. We omit scaling, but the coordinate that remained constant in the latter transformation can be given the value of any of the coordinates before this transformation. An instance of a resulting figure (but this time for six dimensions) is shown in Figure 10.

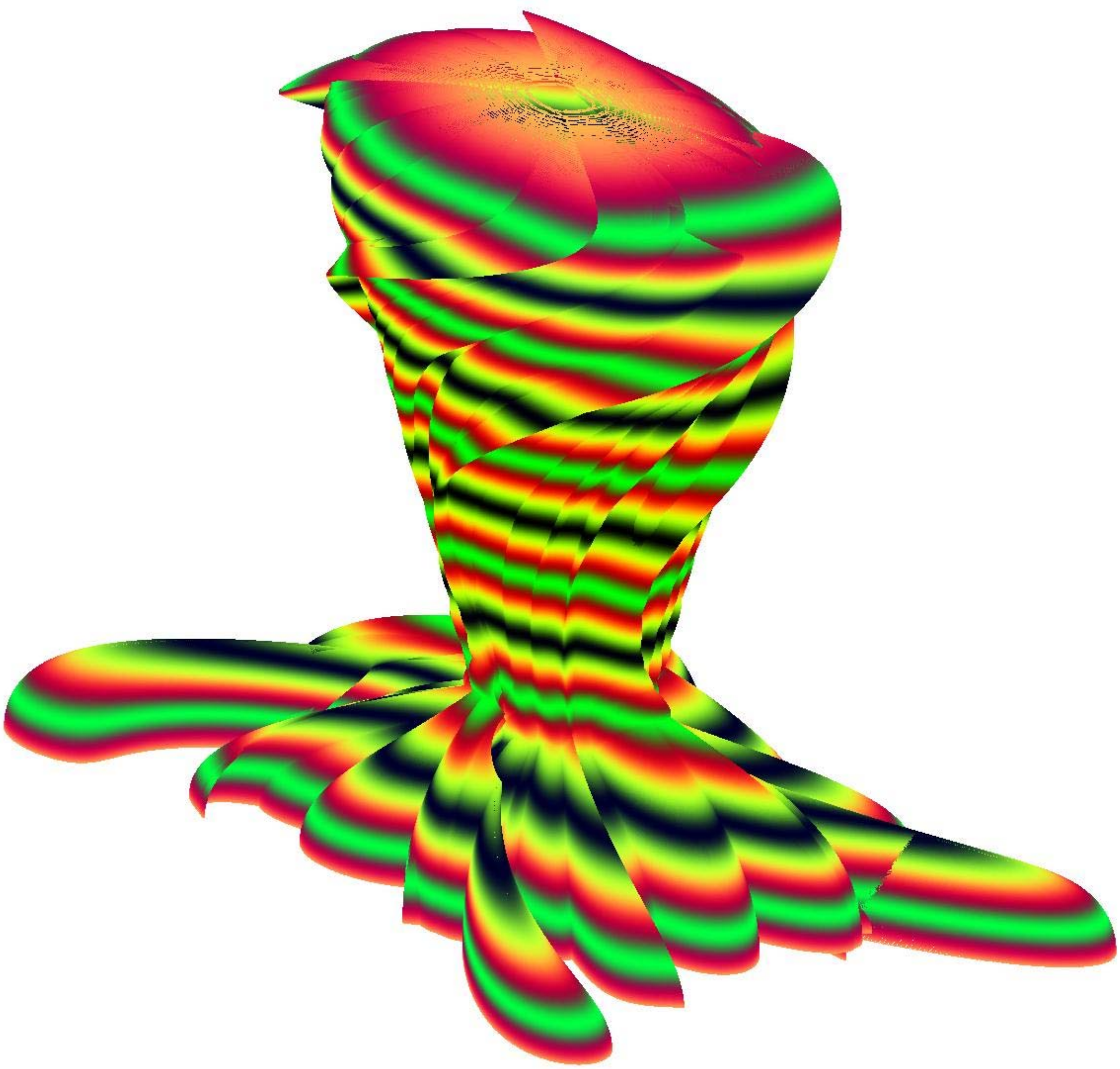

Figure 9. Variation of three dimensional AH(x) for u=(-1,-1,-1,-1,-1,-1,-1,-1)

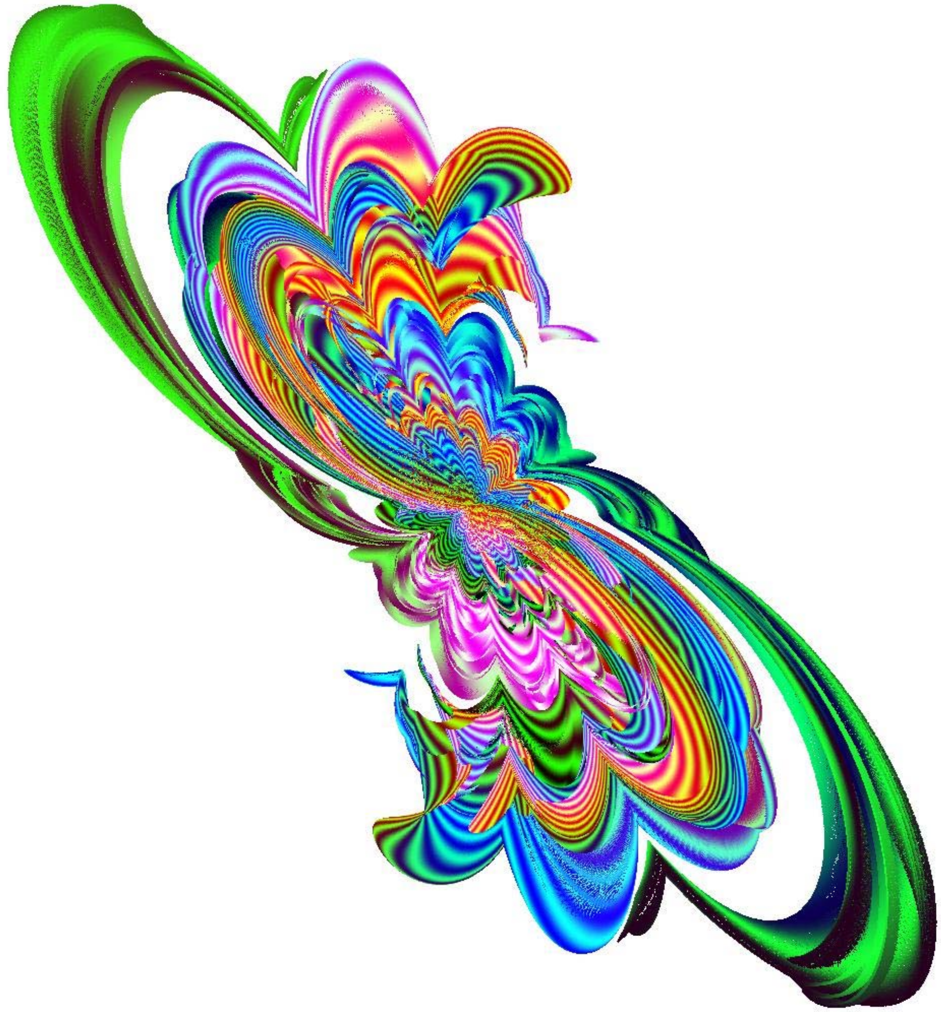

Figure 10. Variation c of three dimensional AH(x) for u=(-1,-1,-1,-1,-1,-1)

4. Intuition, problem solving and the heart transform

Points of a high-dimensional space can be mapped on forms. Consequently, sets of points can be mapped on sets of forms. Suppose that, for instance in the context of a neural network training context, a set of examples is given that belongs to a class. Such examples can be interpreted as points of a high-dimensional space. Then, the form consisting of the union of the individual forms corresponding to the examples can be constructed. Suppose that the forms of the examples are constructed in two dimensions. Since the latter forms tend to overlap each other, one can add a third dimension, or use color codes to depict places where overlap is present. Here, we only illustrate the second option. Further, in case classes consist of large numbers of instances, one can suffice with the contours of forms instead of using entirely filled forms. Consider an 8-parity problem. Such a problem classifies a binary 8-dimensional input-

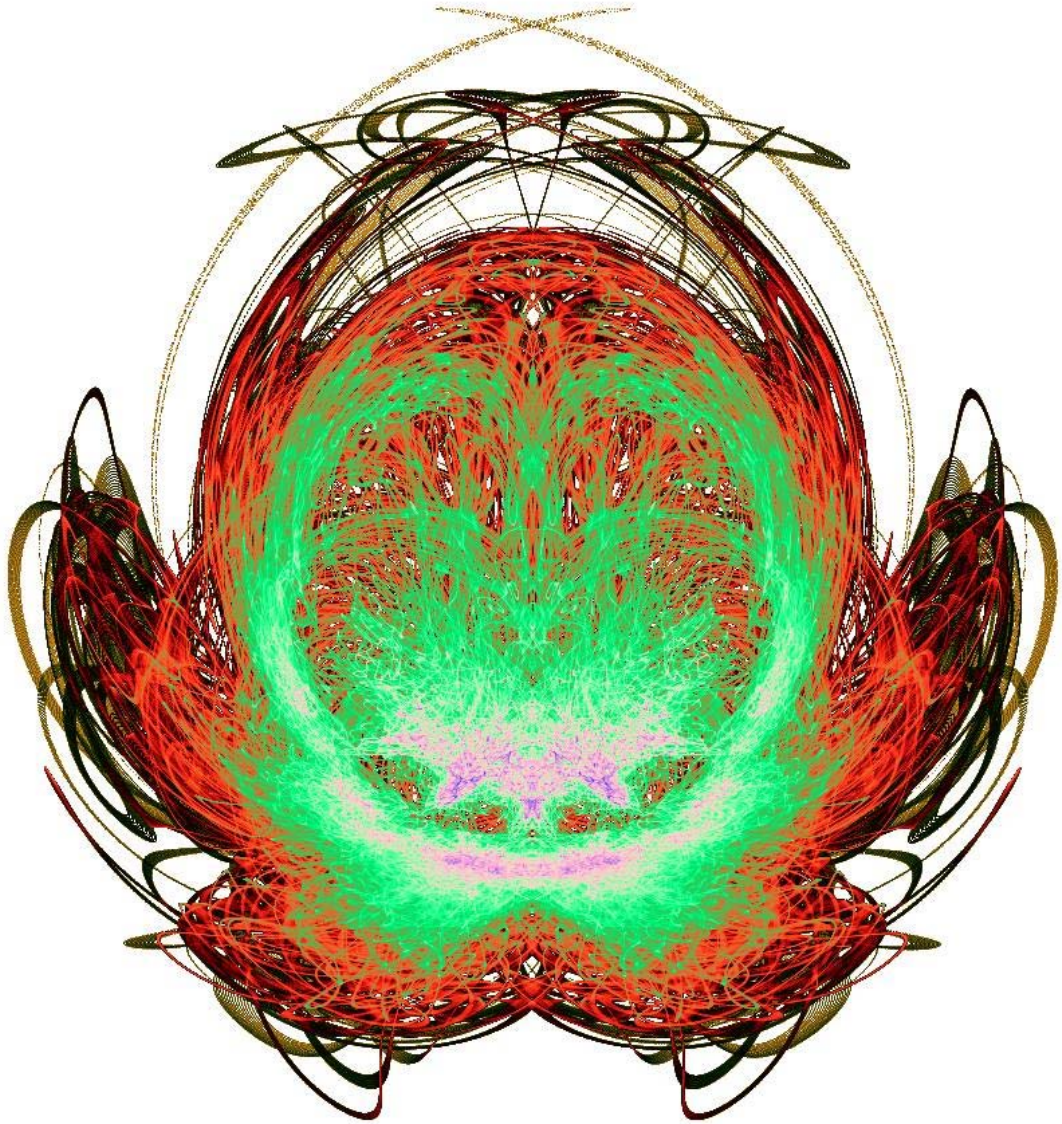

Figure 11. Set of even-parity items for an 8-dimensional input space

space (with components –1 and +1) in two classes [15]. Figure 11 shows the form corresponding to the entire class of training items with even parity (the entire form was rotated over $\pi/4$). Figure 12 shows the set of all training items for a linear problem (more specifically, all binary 8-dimensional items with at most three components equal to +1 were included in the set that is depicted).

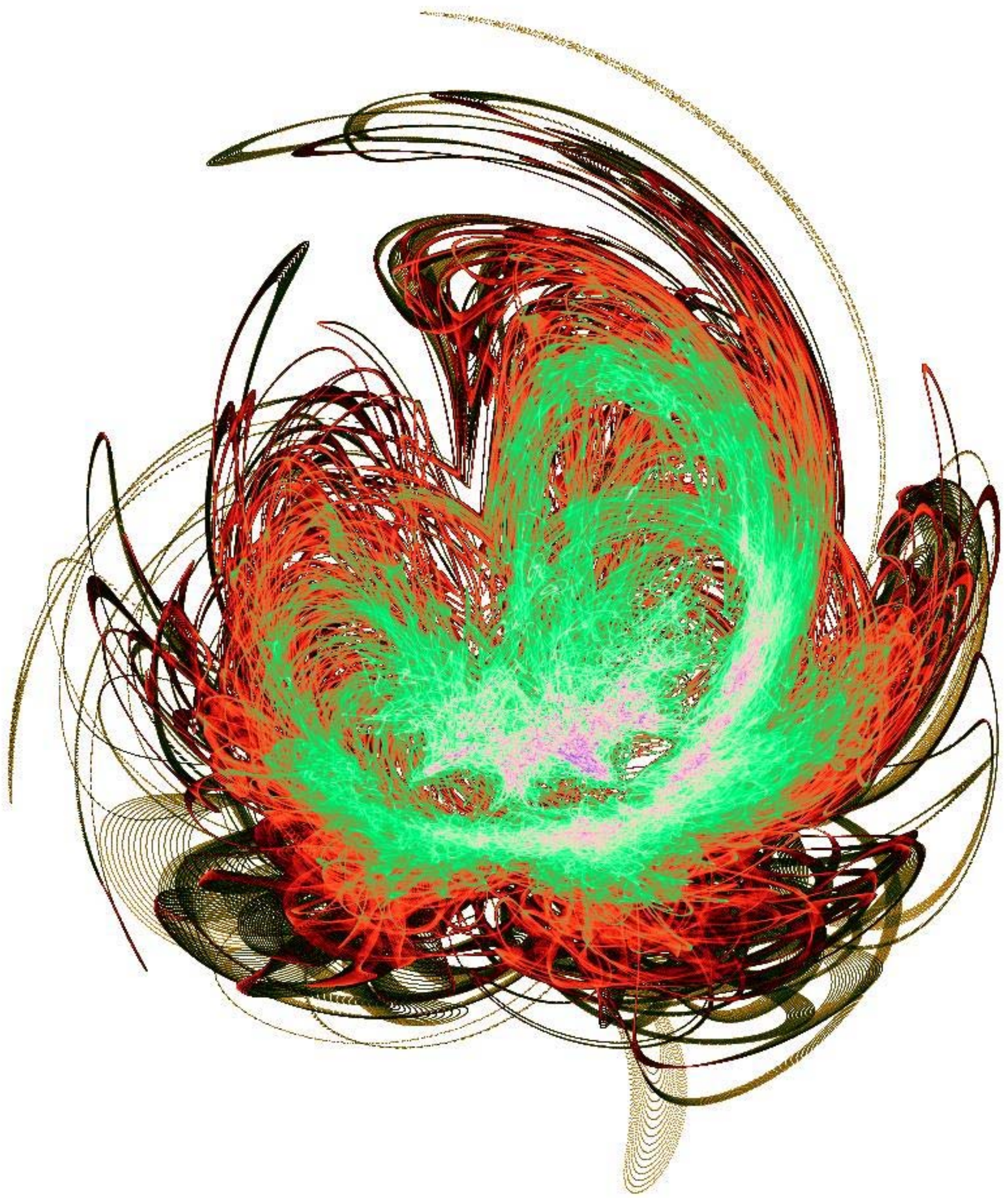

Figure 12. Set of items with sum of +1-components strictly less than four

Figures 11 and 12 illustrate that sets corresponding to classes with demarcations corresponding to benchmark problems lead to aesthetically attractive forms. A class with randomly sampled elements does not show this property. This opens the possibility that, for a suitable mapping, aesthetic criteria can be used to:

(i) identify classes with 'nice' demarcation, even if the algorithmic basis of the demarcation is not known. Figures 11-12 illustrate that classes of both linear and (even very) non-linear algorithmic definition correspond to representations of visual elegancy

(ii) complete a class of examples. Suppose, for instance, that a subset of even parity items is given. Then, by looking at the effect on the visual representation of including other items, it can be decided if the original set is enlarged with the other items or not.

We notice that properties (i) and (ii) are beyond what is possible in neural network or related contexts. Pattern recognition is very well possible by such methods, but only if a large number of instances is given on the beforehand. Here, interesting classes suggest themselves even before any training with stimuli coming from an outside world. This is another way in which the present approach reminds one of Penrose's (neo)platonic view [13]. Further, the symmetries at the basis of the perception of beauty in the present type of visualizations are not easily defined (except for the mirror symmetry in Figure 11). It may be pretty hard (or even impossible) to write an algorithm that is able to identify all symmetries at the basis of the experience of beauty for the present type of forms. This non-algorithmic feature figures prominently in Penrose's theory. More illustrations, and a more elaborate argumentation of this point can be found in [15].

5. Conclusion

The present approach to generative design is complementary to the well known genetic generative design approach of Soddu [15]. The latter works with smallest elements that are assembled into an aesthetically attractive whole. Here, use is made of a movement that can be described on an intuitive level as a change comparable to a Fourrier transform. The parameters of the present class of forms refer to the whole of the forms, not to localizable small elements. Nevertheless, such parameters can be included in genetic evolution schema's. Much work remains to be done. We mention:

-A more solid mathematical study and justification of the choice of AH(x) and the study of its variations

-Figure 10 was included because, when seen from an appropriate perspective, it reminds of a chair; the extension and use of the present class of forms has to be investigated

-Steps toward concrete links to architecture have to be made; the technology to realize organic constructs in practice has to be linked to the present type of forms

-The point made on intuition in section 4 is philosophically intriguing; it should be tested in terms of empirical aesthetics if humans are able to demarcate on an aesthetic basis classes that they cannot determine on an algorithmic basis

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